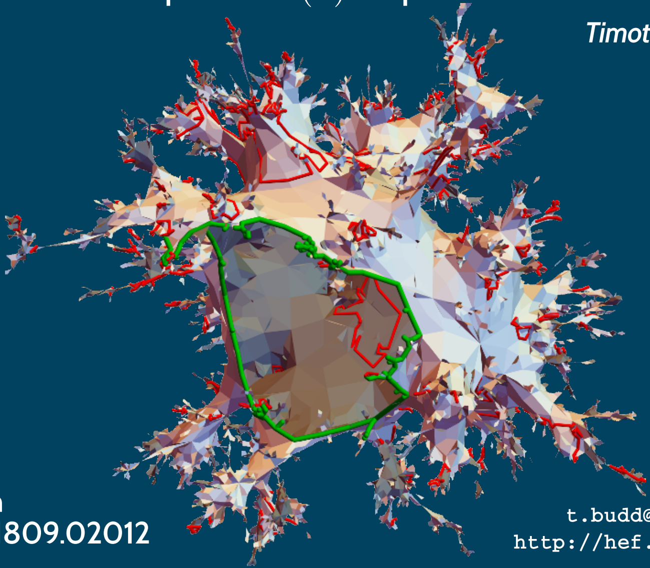
Physics and Mathematics of Discrete Geometries @ Nagoya, Japan - 07-11-2018

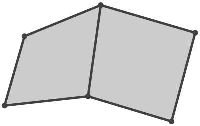
Geometry of two-dimensional quantum gravity coupled to $\mathrm{O}(n)$ loop models

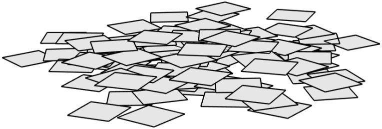
Timothy Budd

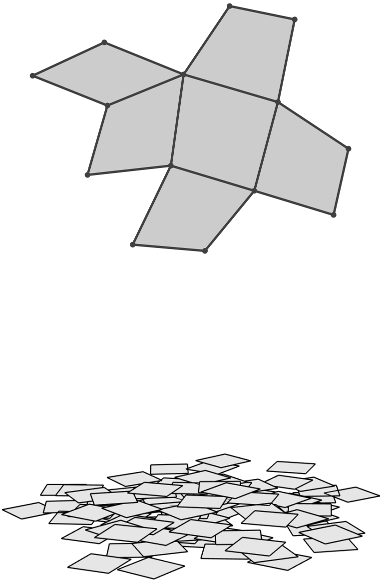


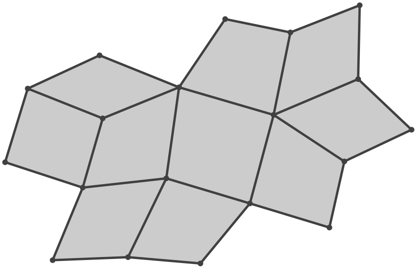
Based on arXiv:1809.02012

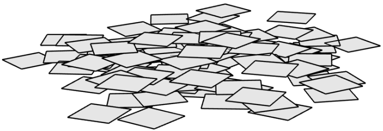
t.budd@science.ru.nl
http://hef.ru.nl/~tbudd/

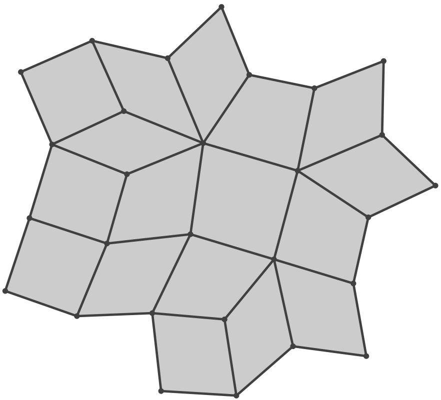


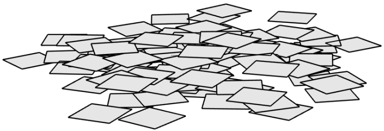


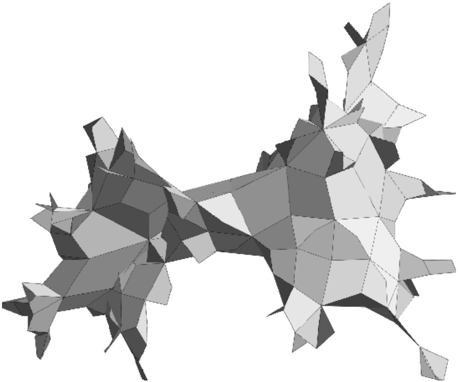


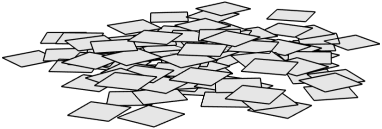




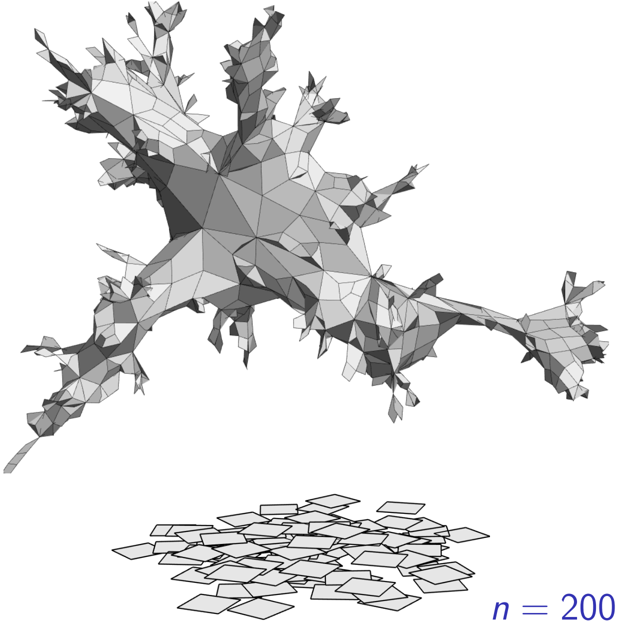




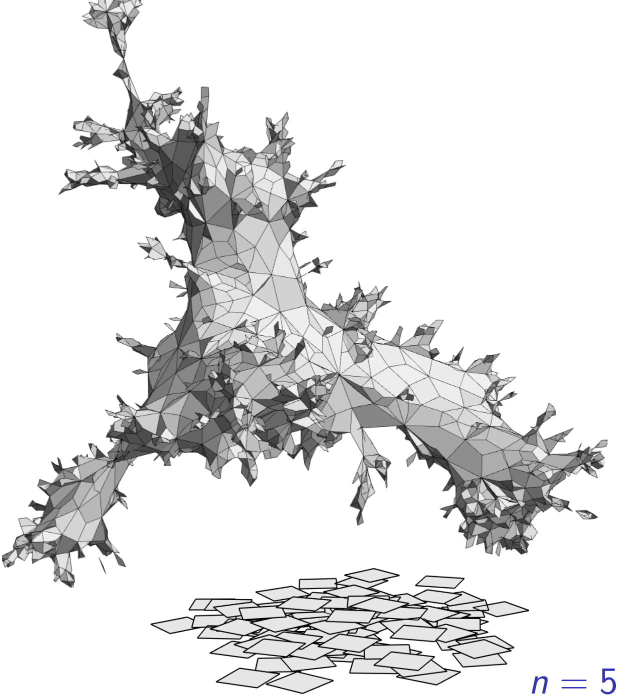




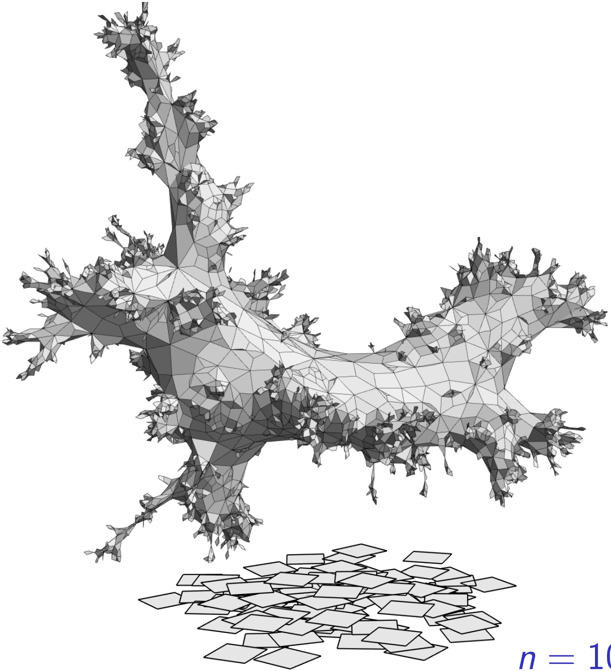
$n = 200$



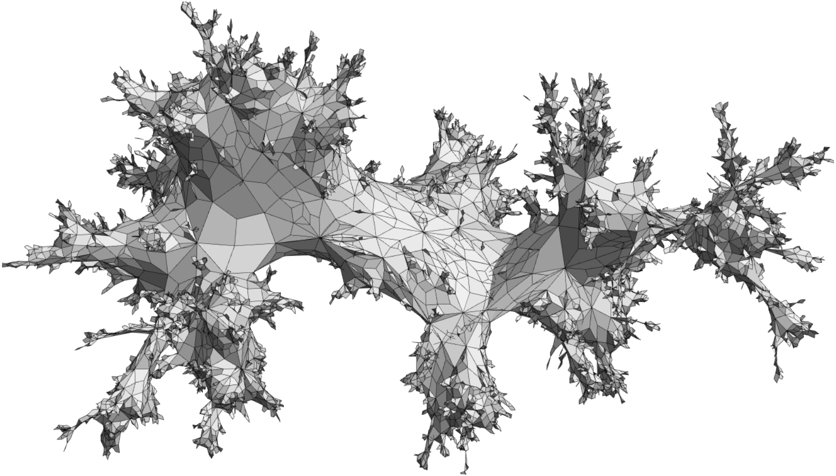
$n = 2000$

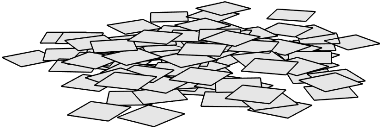


$n = 5000$

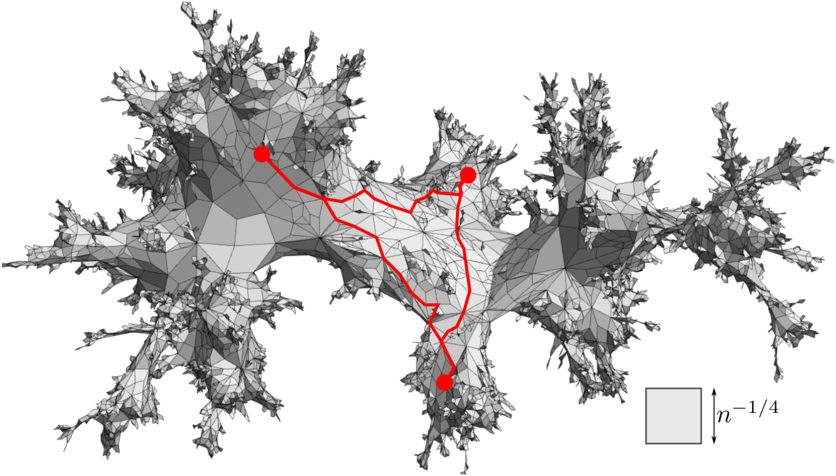


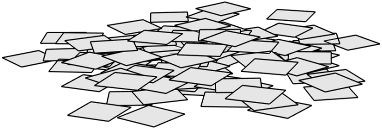
$n = 10000$





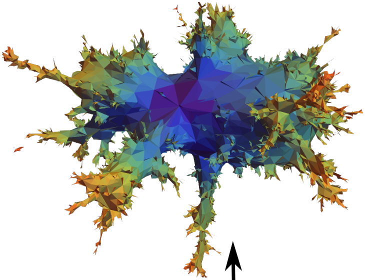
$n = 20000$





$n = 20000$

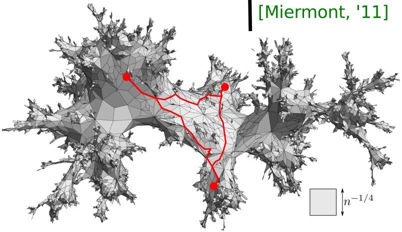
Brownian sphere



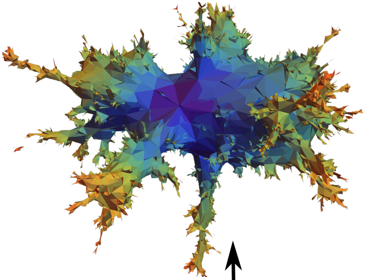
$n \to \infty$

Convergence in distribution
w.r.t. Gromov-Hausdorff
topology on metric spaces

[Le Gall, '11]
[Miermont, '11]



Brownian sphere



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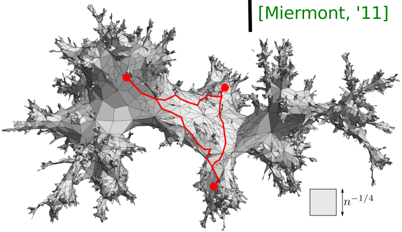
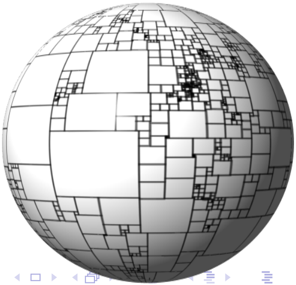


Uniformization of
Riemannian metric



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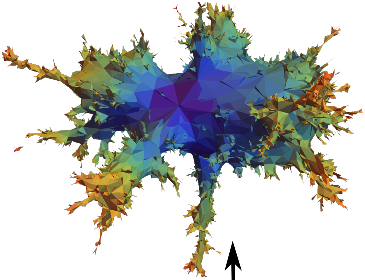
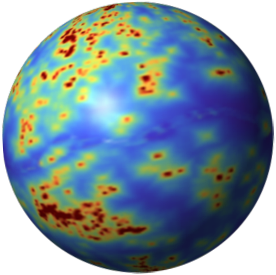


LQG$_{\gamma=\sqrt{8/3}}$



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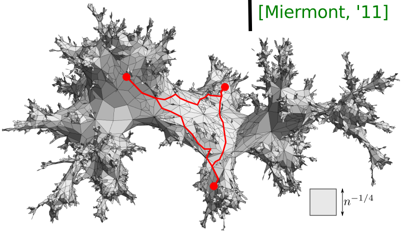
topology on metric spaces

[Le Gall, '11]
[Miermont, '11]

$n \to \infty$

Conjectured
convergence of
measure on S^2



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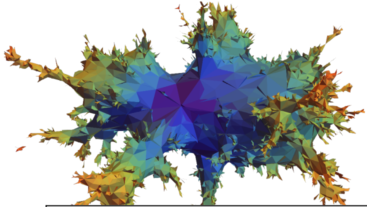

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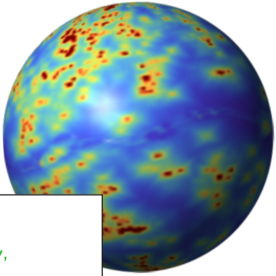


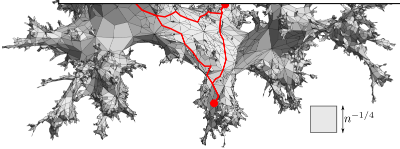
- Liouville Quantum Gravity (LQG_γ) [Polyakov, Knizhnik, Zamolodchikov, David, Distler, Kawai, ..., '80s]

  Conformal field theory of the conformal mode ϕ

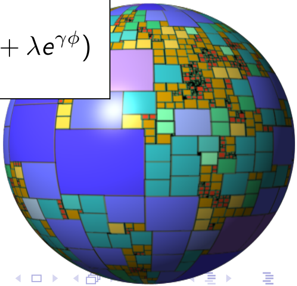
$$\gamma \in [0,2], \qquad g_{ab} = e^{\gamma\phi}\hat{g}_{ab}$$

$$S[\phi] = \frac{1}{2}\int \mathrm{d}^2 z\sqrt{\hat{g}}(\hat{g}^{ab}\partial_a\phi\partial_b\phi + Q_\gamma \hat{R}\phi + \lambda e^{\gamma\phi})$$

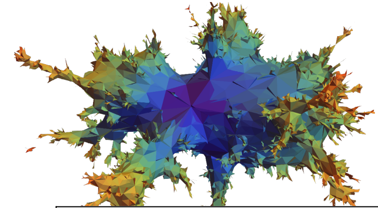
Conjectured convergence of measure on S^2



Uniformization of Riemannian metric



Brownian sphere



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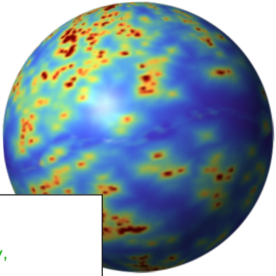

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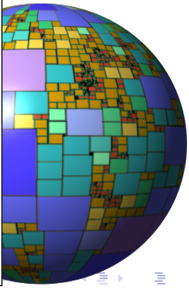

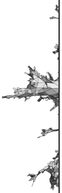

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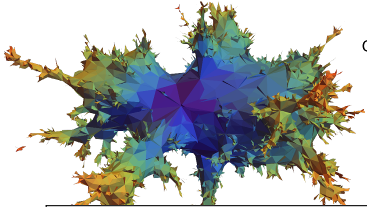
- Probabilistic interpretation: regularization $\phi \to \phi_\epsilon$ yields a well-defined random measure

$$\sqrt{\hat{g}}e^{\gamma(\phi_\epsilon - \mathbb{E}\phi_\epsilon^2/2)}\mathrm{d}^2 z \xrightarrow{\epsilon\to 0} \mathrm{d}\mu_{\mathrm{LQG}}$$

[Duplantier, Sheffield, '08] [Kahane, '85] [David, Kupiainen, Rhodes, Vargas, '14]

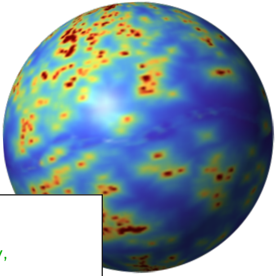
Brownian sphere



Quantum Loewner Evolution
[Miller, Sheffield, '16]

Uniformization
[Gwynne, Miller, Sheffield, '18]

$\mathrm{LQG}_{\gamma=\sqrt{8/3}}$



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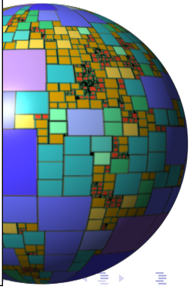

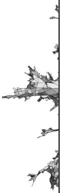

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Brownian sphere

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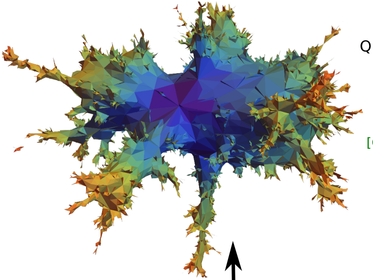

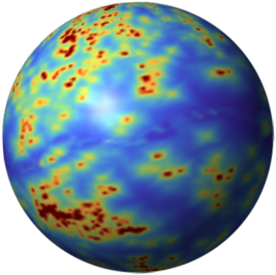

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$n \to \infty$

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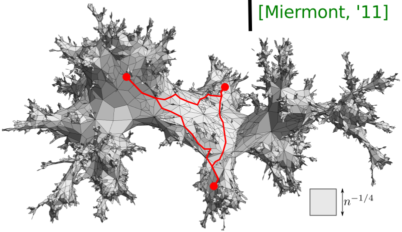
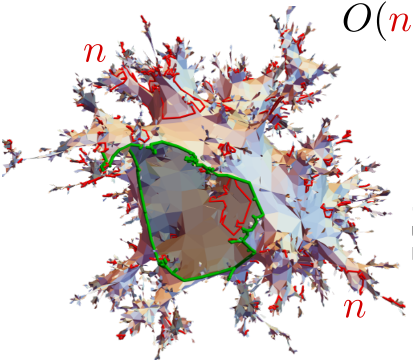
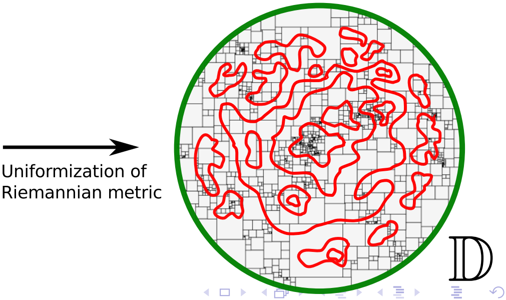
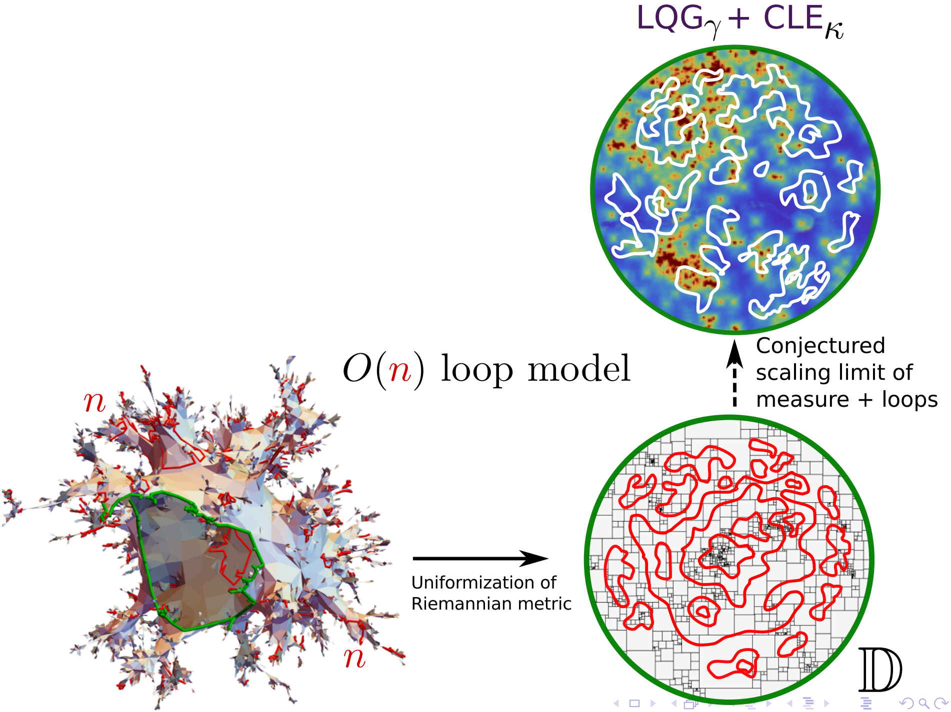


Uniformization of
Riemannian metric

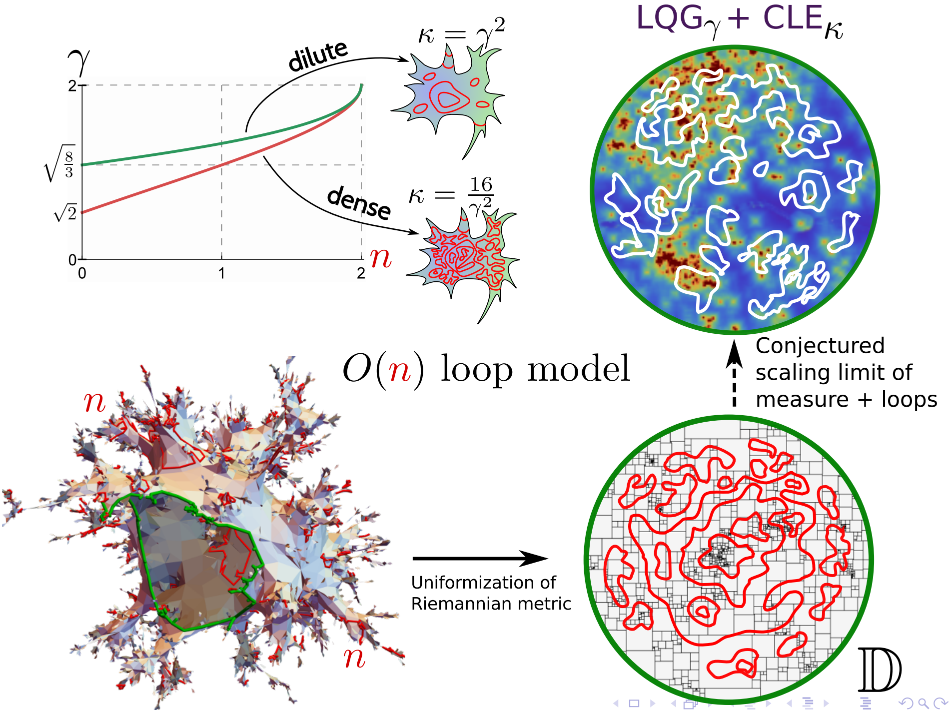
$O(n)$ loop model



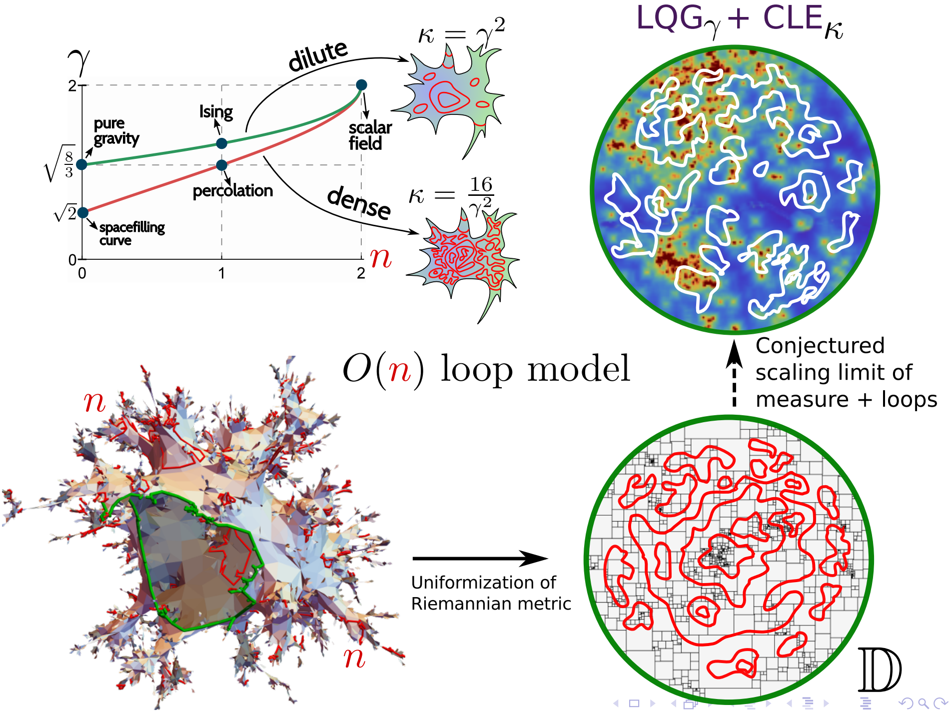




$LQG_\gamma + CLE_\kappa$
Conjectured scaling limit of measure + loops
$O(n)$ loop model
n
n
Uniformization of Riemannian metric
$\mathbb{D}$



$\mathrm{LQG}_\gamma + \mathrm{CLE}_\kappa$
γ
2
$\sqrt{\frac{8}{3}}$
$\sqrt{2}$
0
0
1
2
n
dilute
$\kappa = \gamma^2$
dense
$\kappa = \frac{16}{\gamma^2}$
$O(n)$ loop model
Conjectured scaling limit of measure + loops
n
n
Uniformization of Riemannian metric
$\mathbb{D}$


$\mathrm{LQG}_\gamma + \mathrm{CLE}_\kappa$
γ
2
$\sqrt{\frac{8}{3}}$
$\sqrt{2}$
0
0
1
2
n
pure gravity
Ising
scalar field
percolation
spacefilling curve
dilute
dense
$\kappa = \gamma^2$
$\kappa = \frac{16}{\gamma^2}$
$O(n)$ loop model
n
n
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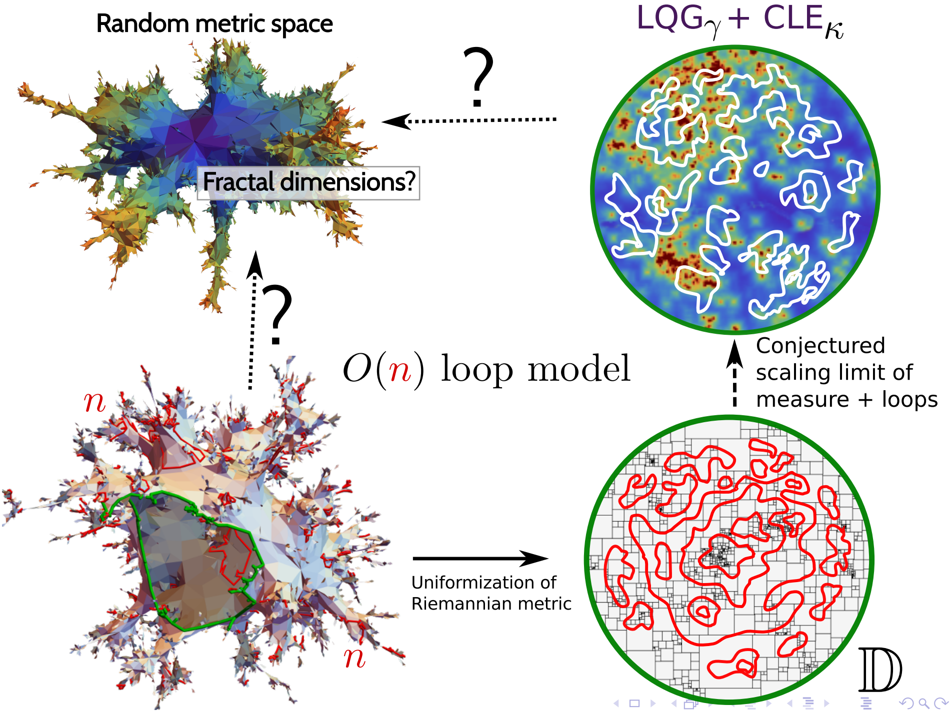
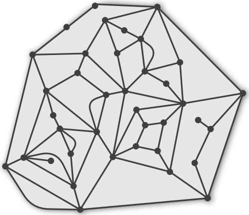

Random metric space
$\mathrm{LQG}_\gamma + \mathrm{CLE}_\kappa$
?
Fractal dimensions?
?
$O(n)$ loop model
Conjectured
scaling limit of
measure + loops
n
n
Uniformization of
Riemannian metric
$\mathbb{D}$

Planar maps coupled to a rigid $O(n)$ loop model

- Planar map: planar (multi)graph properly embedded in $\mathbb{R}^2$ viewed up to continuous deformations.



Planar maps coupled to a rigid $O(n)$ loop model

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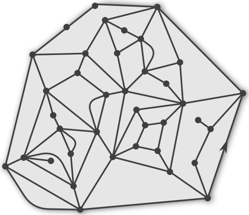

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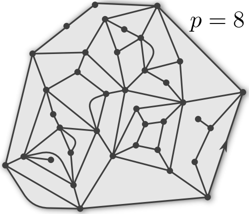
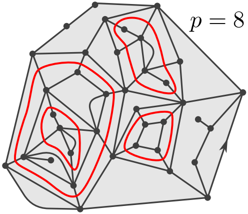


Planar maps coupled to a rigid $O(n)$ loop model

- Planar map: planar (multi)graph properly embedded in $\mathbb{R}^2$ viewed up to continuous deformations. Rooted, perimeter $2p$ fixed. Bipartite for simplicity.
- $O(n)$ loop model: add disjoint loops that intersect quadrangles rigidly. Partition function $W^{(p)} = \sum_{\mathfrak{m} \text{ of perim } 2p} w_{n,g,\mathbf{q}}(\mathfrak{m})$,

$w_{n,g,\mathbf{q}}(\mathfrak{m}) = n^{\#}$  $g^{\#}$  $\prod_{\text{reg. faces } f} q_{\frac{\deg(f)}{2}}$

for $n, g, q_1, q_2, q_3, \ldots \in \mathbb{R}_+$ fixed.



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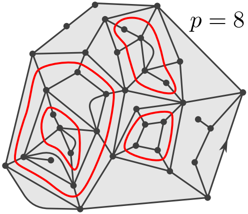
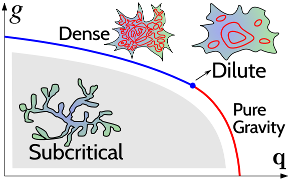


$$w_{n,g,\mathbf{q}}(\mathfrak{m}) = n^{\#\circ} g^{\#\square} \prod_{\text{reg. faces } f} q_{\frac{\deg(f)}{2}}$$

for $n, g, q_1, q_2, q_3, \ldots \in \mathbb{R}_+$ fixed.

- For $n \in (0,2)$ the model has four phases as $p \to \infty$: [Borot, Bouttier, Guitter, '11] [TB, Chen, '18]
  - subcritical: treelike/only see boundary
  - pure gravity: microscopic loops
  - dilute critical: self-avoiding loops
  - dense critical: self-touching loops



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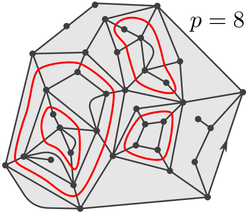

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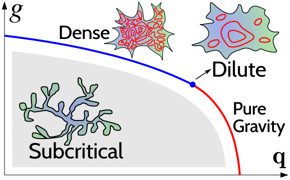
$$w_{n,g,\mathbf{q}}(\mathfrak{m}) = n^{\#} \quad g^{\#} \quad \prod_{\text{reg. faces } f} q_{\frac{\deg(f)}{2}}$$

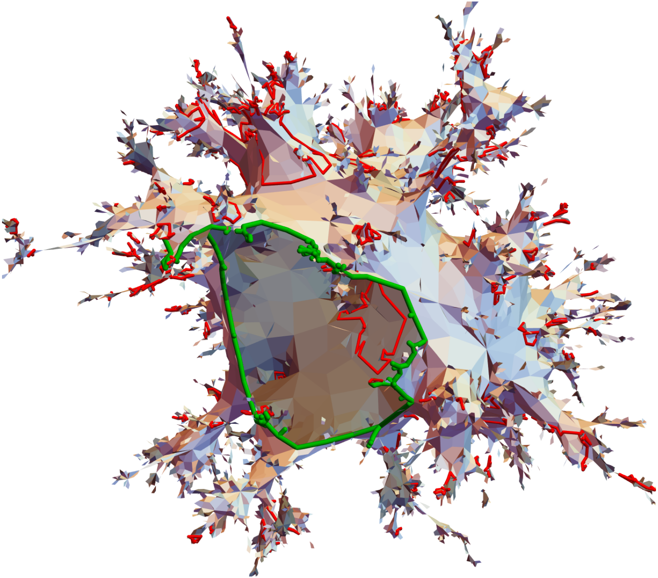
 

for $n, g, q_1, q_2, q_3, \ldots \in \mathbb{R}_+$ fixed.

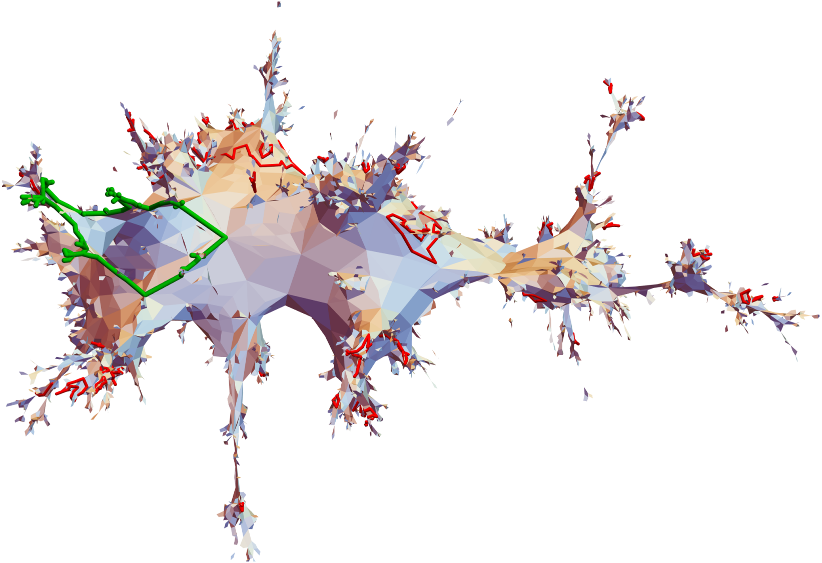
- For $n \in (0,2)$ the model has four phases as $p \to \infty$: [Borot, Bouttier, Guitter, '11] [TB, Chen, '18]
  - subcritical: treelike/only see boundary
  - pure gravity: microscopic loops
  - dilute critical: self-avoiding loops
  - dense critical: self-touching loops



Simulation: dilute quadrangulation ($q_2 > 0, q_1 = q_3 = \ldots = 0$), $p = 50$, $n = 0.6$



Simulation: dilute quadrangulation ($q_2 > 0, q_1 = q_3 = \ldots = 0$), $p = 40$, $n = 0.3$



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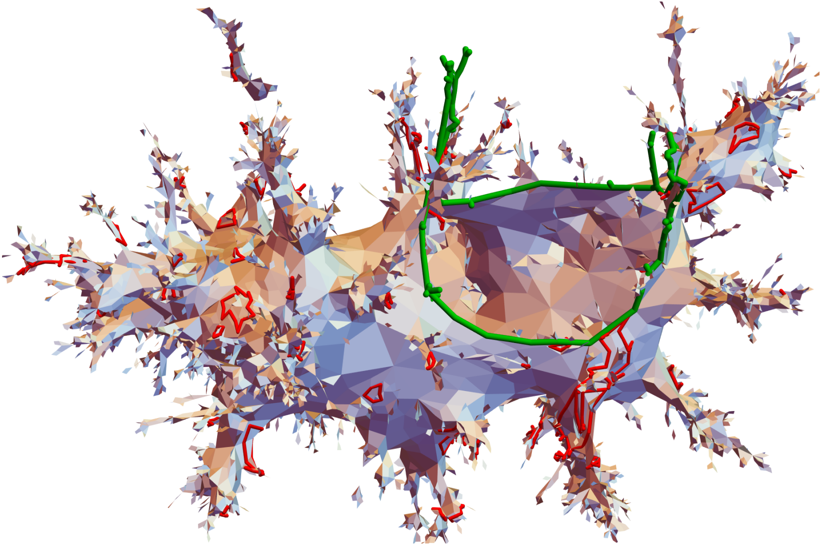
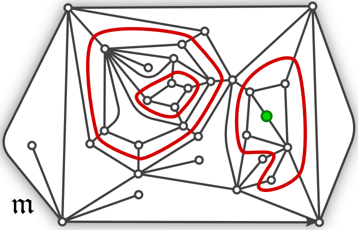


First-passage percolation

- Assign i.i.d. $\mathrm{Exp}(1)$ lengths to dual edges, but 0 to loop-edges.



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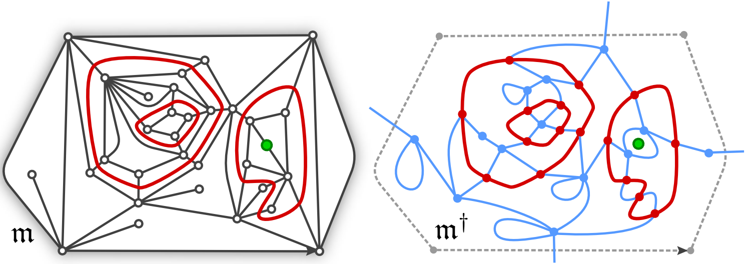

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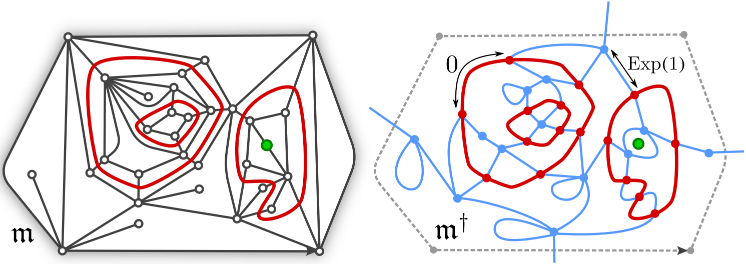

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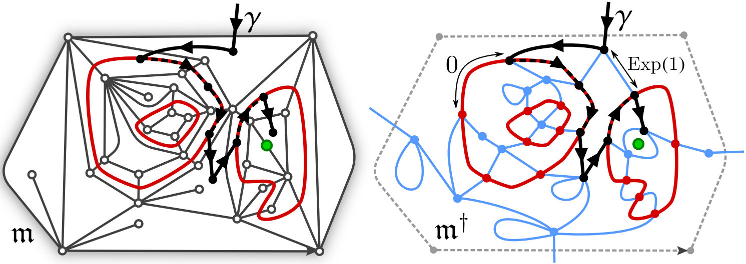

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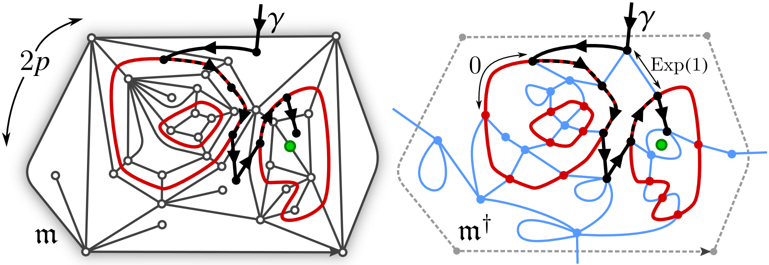

Theorem (TB, '18)

In the dilute phase the fpp-distance d_{fpp} between the boundary and a random vertex of a loop-decorated map of perimeter 2p satisfies:

$$\frac{d_{fpp}}{c\,p^b} \xrightarrow[p\to\infty]{(d)} R, \qquad b = \frac{1}{\pi}\arccos\left(\frac{n}{2}\right) \in \left(0, \frac{1}{2}\right],$$

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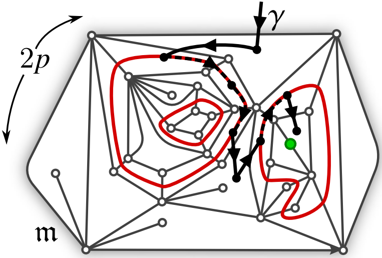

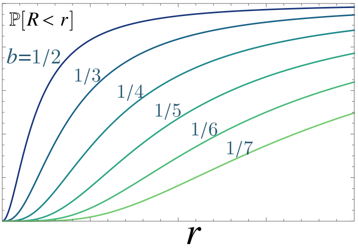

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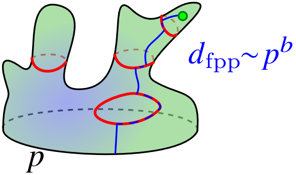

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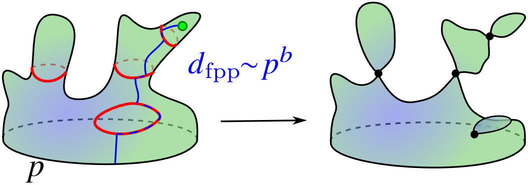

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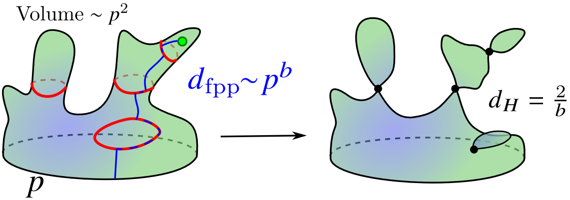

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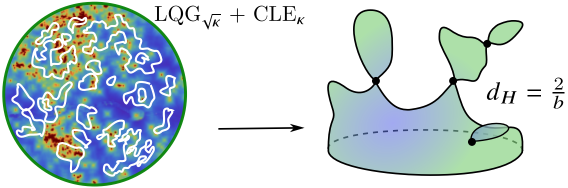

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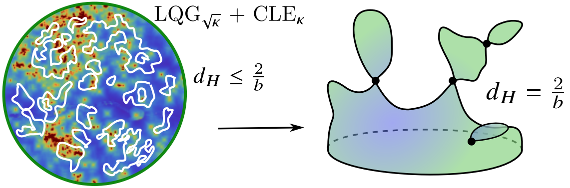
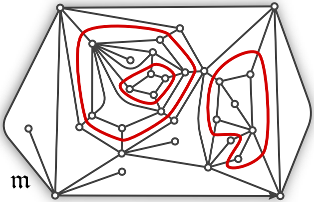
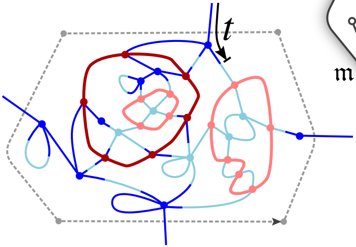


- Define the ball of radius t of $\mathfrak{m}$:





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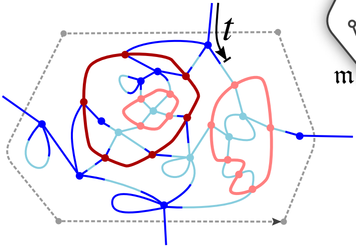

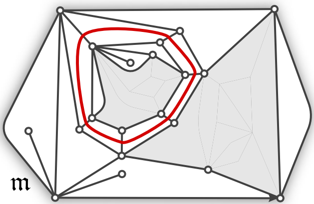

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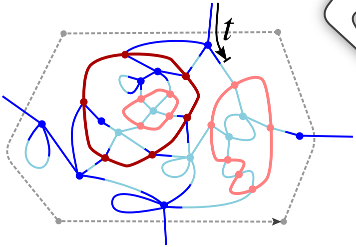

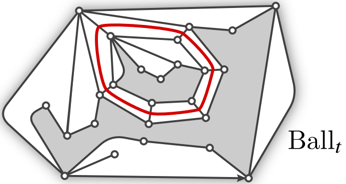

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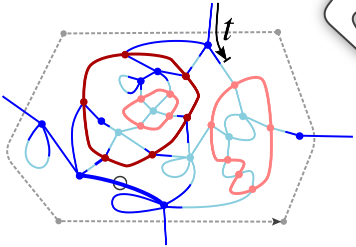

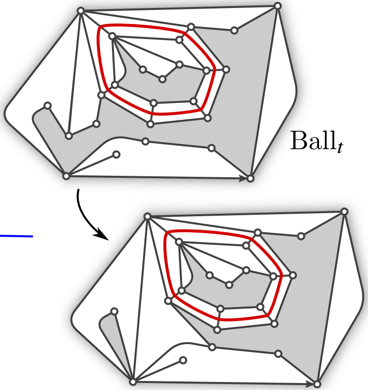

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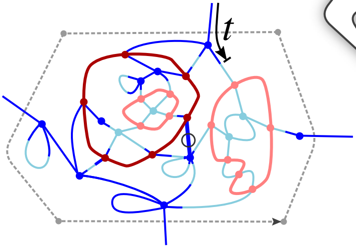

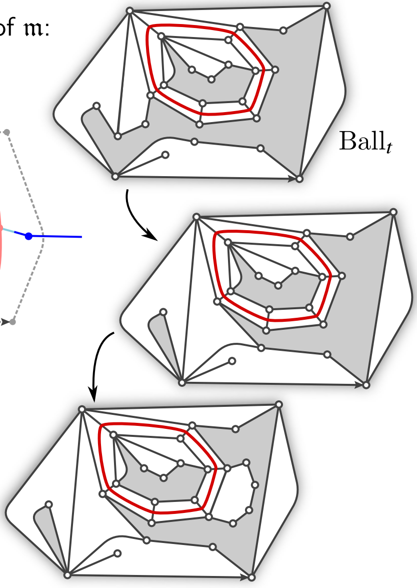

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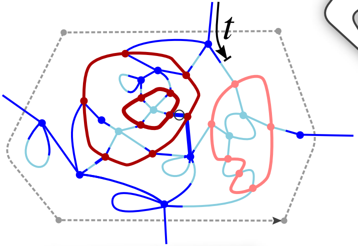

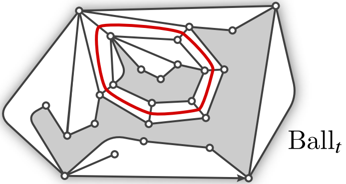

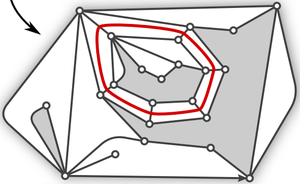

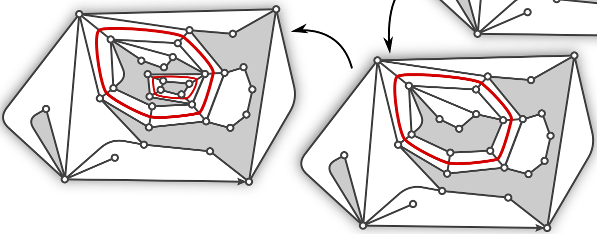

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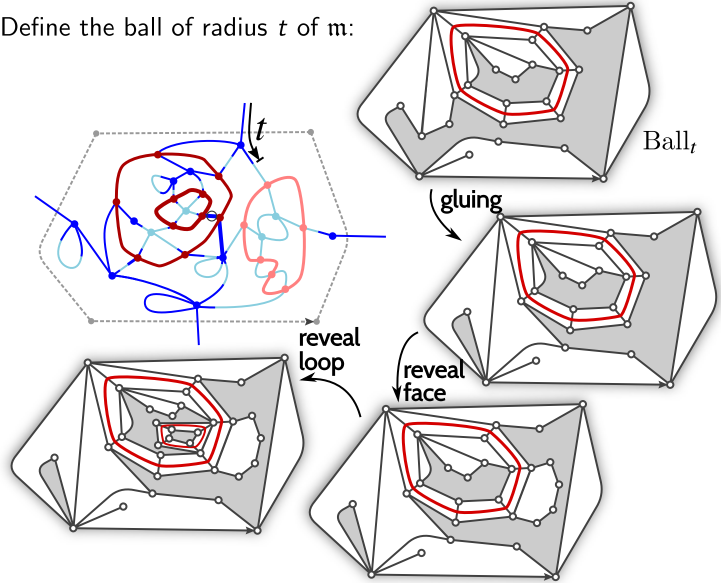

- Example of a peeling exploration of $\mathfrak{m}$ with three types of events.

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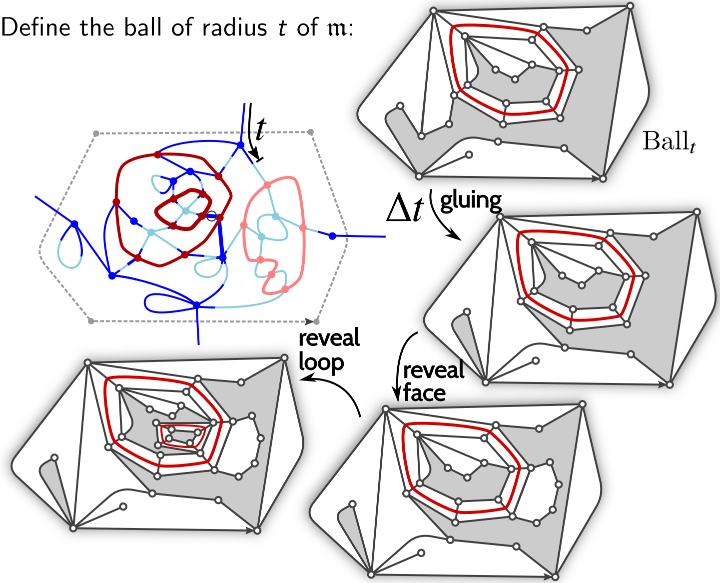
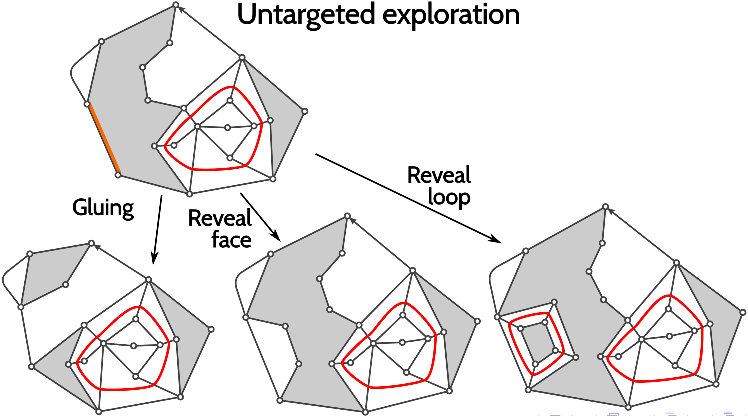


- Example of a peeling exploration of $\mathfrak{m}$ with three types of events.
- Due to exponential law, events occur uniformly on the hole boundaries, and $\Delta t = \mathrm{Exp}(1/|\text{hole boundaries}|)$.

Peeling exploration

- Introduced in [Watabiki, '95], it led to the first derivation of a distance statistic in random triangulations [Ambjørn, Watabiki, '95].



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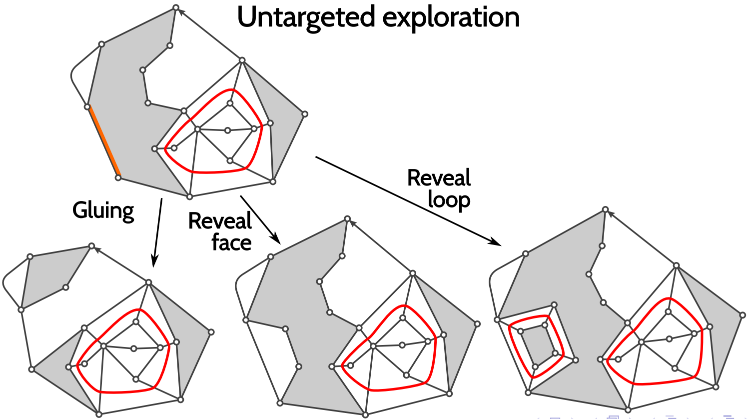

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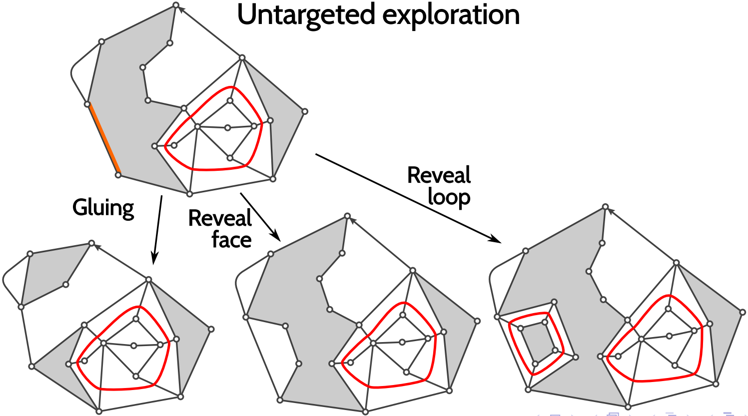

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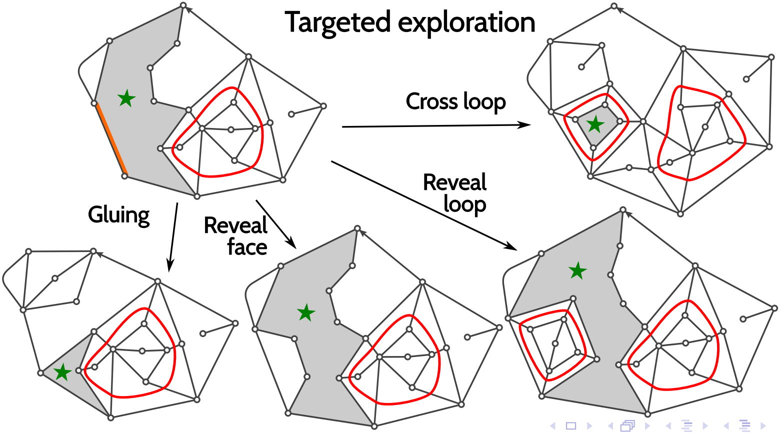
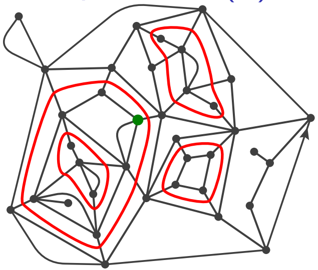
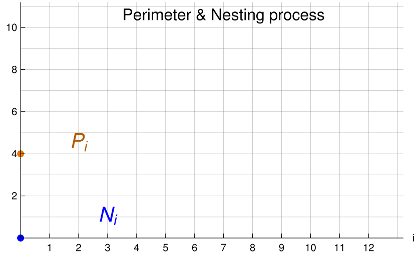


Targeted peeling exploration of map with $O(n)$ model



- Mark a random vertex.



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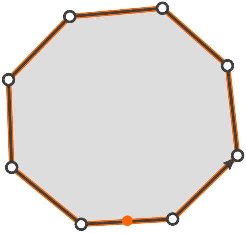

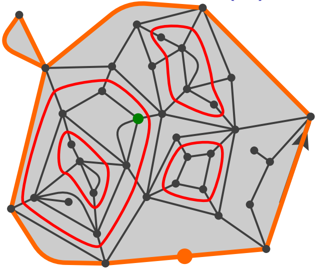

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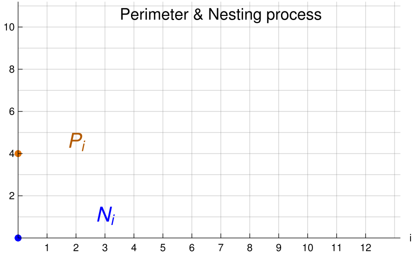

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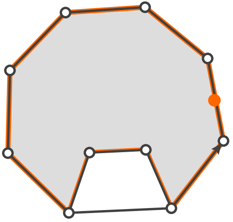

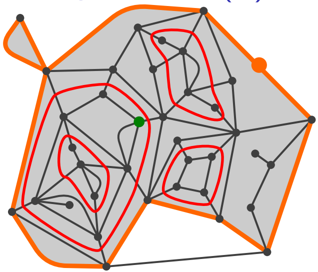

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- Explore by 3 types of events:
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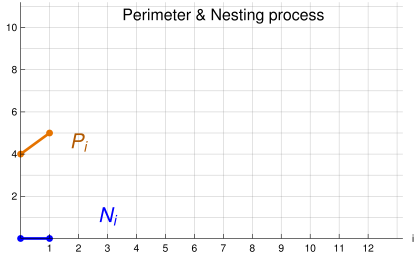
  - Glue pair of edges.
- Track half-length of frontier and # of loops crossed.



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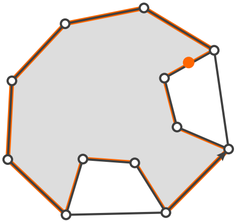

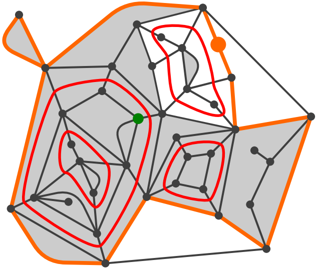

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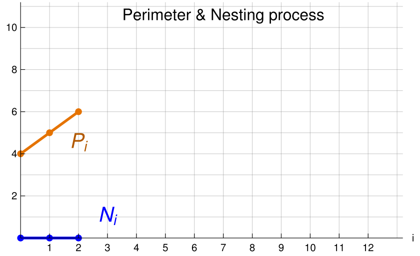

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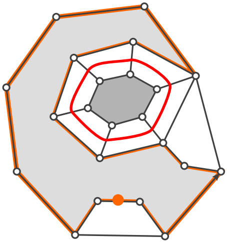

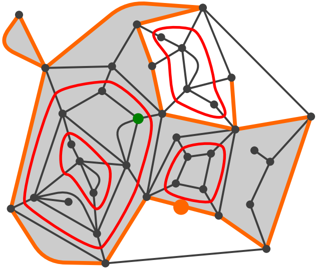

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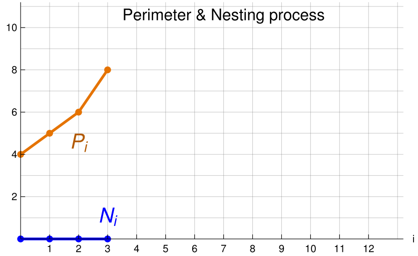

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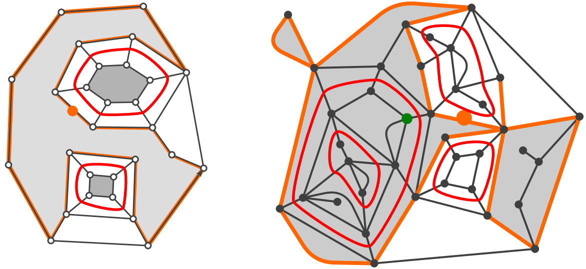

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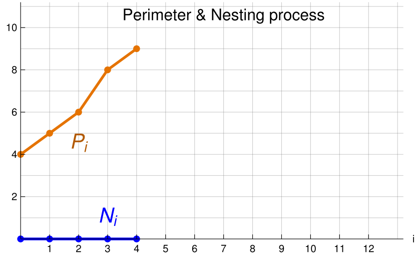

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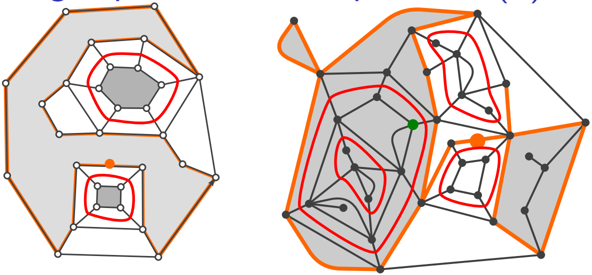

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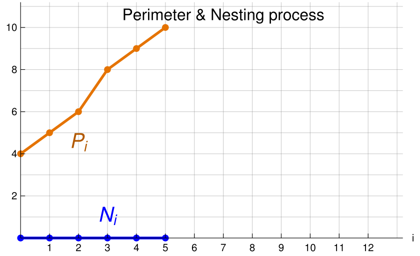

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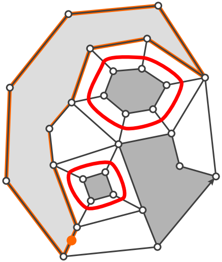

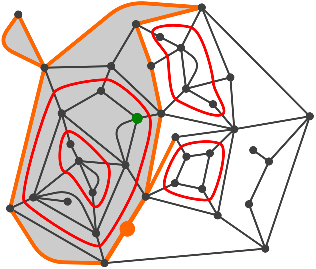

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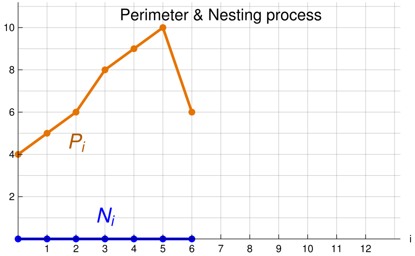

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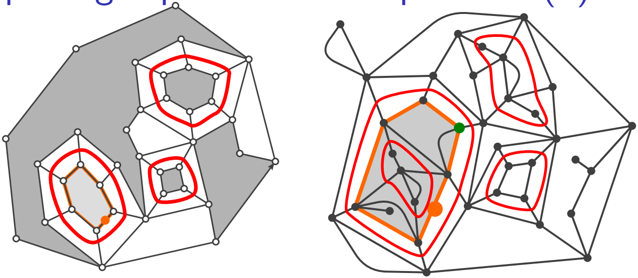

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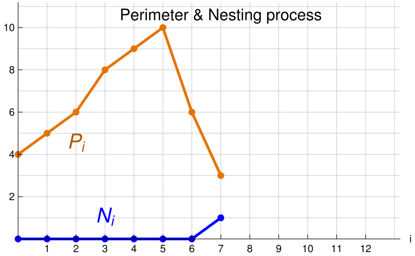

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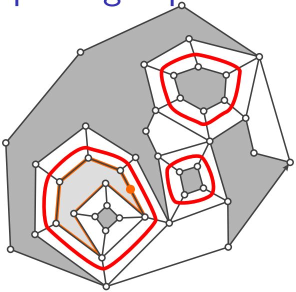

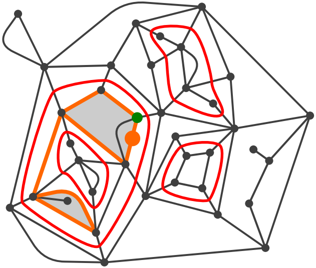

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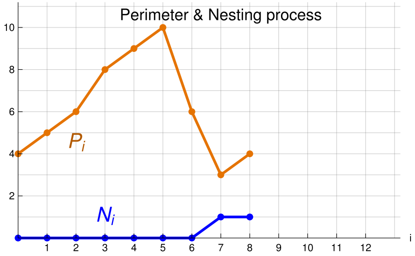

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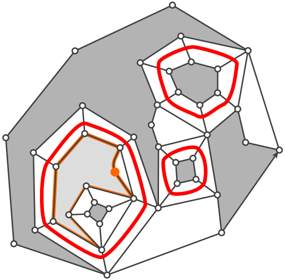

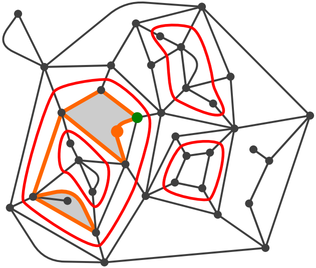

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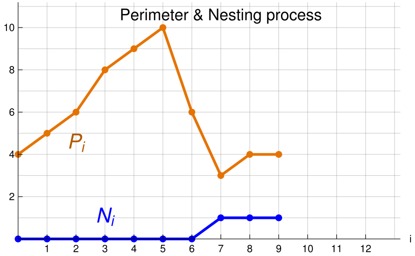

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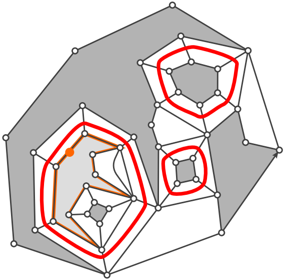

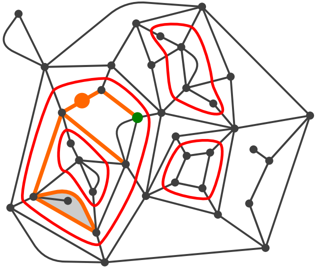

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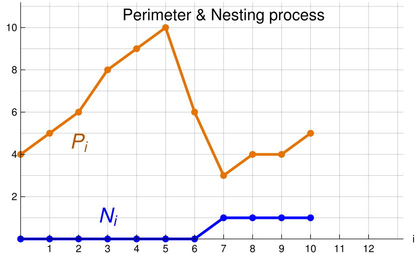

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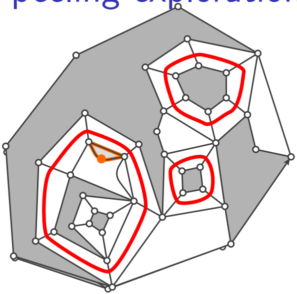

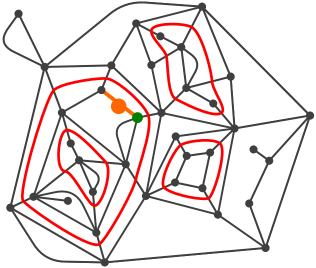

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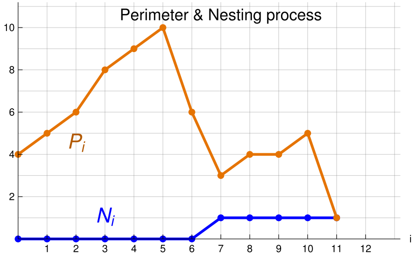

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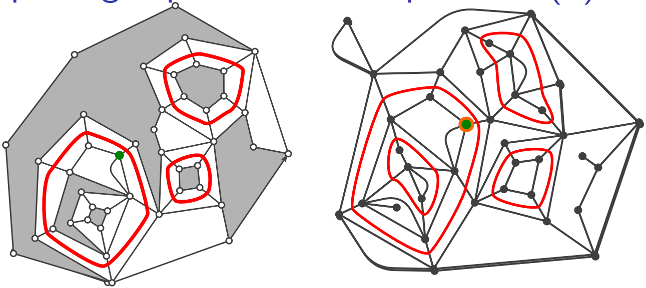

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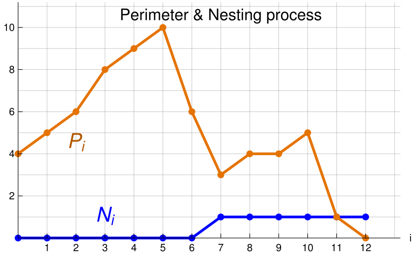

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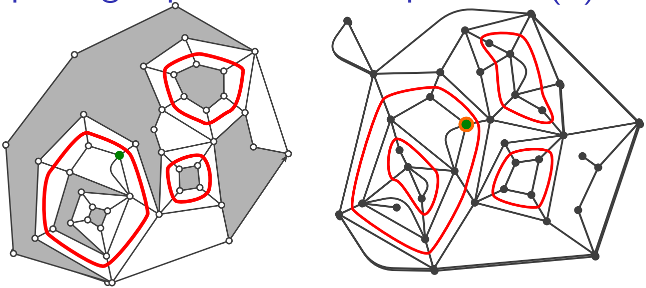

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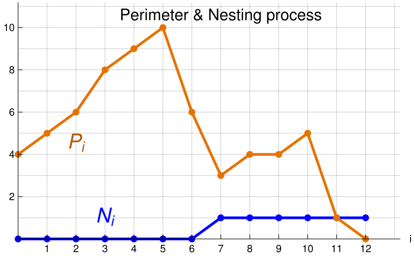
- (P_i, N_i) is a Markov process independent of peel algorithm!



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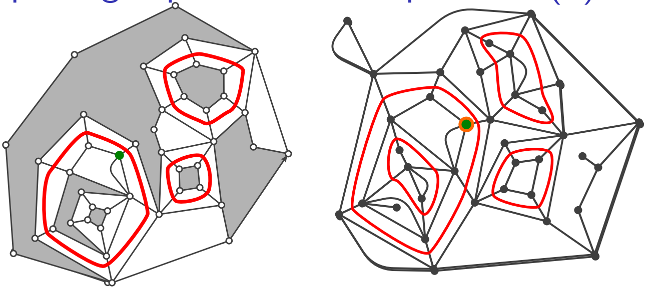

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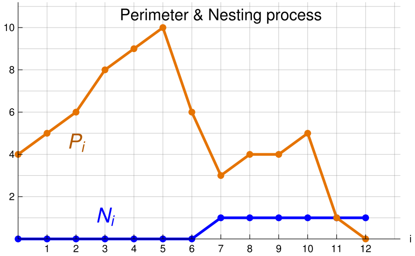
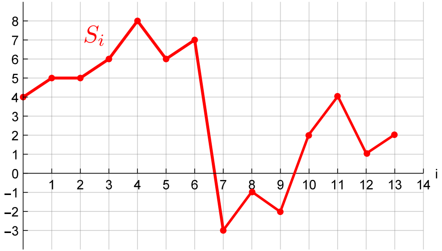


- If we know its law, can compute $d_{\mathrm{fpp}} = \sum_i \frac{\mathrm{Exp}(1)}{2P_i}$.

Ricocheted random walk

- Let (S_i) be a random walk with increments of law $\nu : \mathbb{Z} \to [0,1]$.



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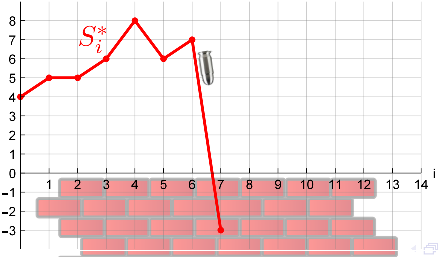
- For $\mathfrak{p} \in [0,1]$, define $\mathfrak{p}$-ricocheted random walk (S_i^*):



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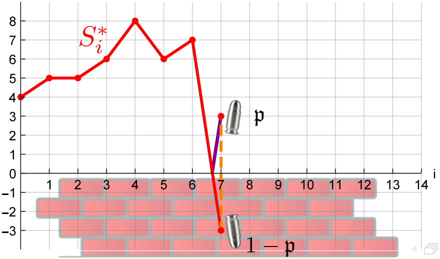

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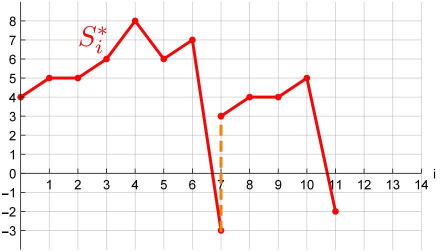

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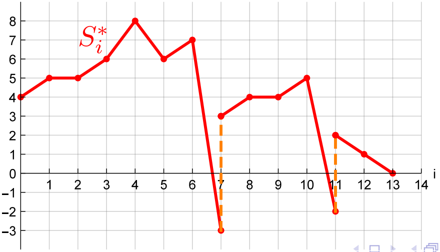

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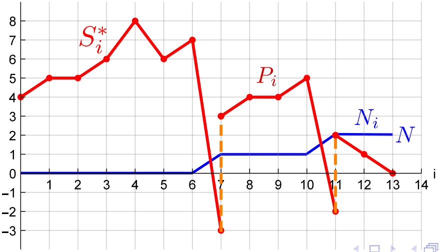

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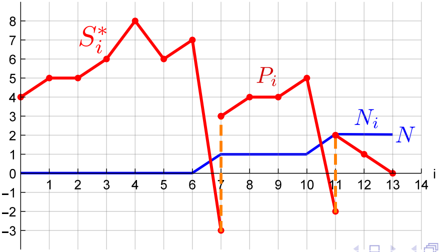
Proposition (TB,'18)

For $(\mathbf{q}, g, n)$ in the dilute phase: there exists a law ν such that

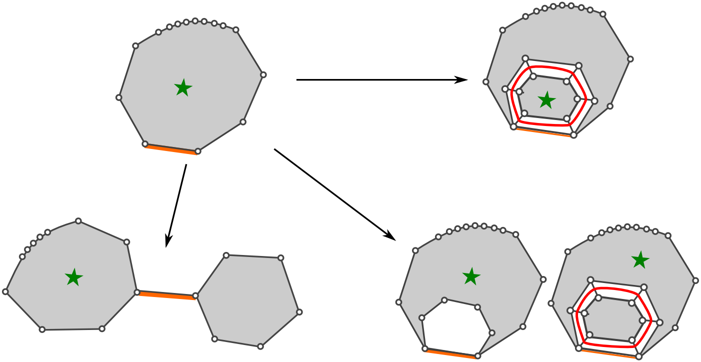
$(P_i, N_i) \overset{(\mathrm{d})}{=} (S_i^*, \#\text{ricochets})$ conditioned to be absorbed at 0, with

$$\mathfrak{p} = \frac{n}{2}, \quad \nu(k) = \begin{cases} g^{-k} q_{k+1} + n\, g^{k+2} W^{(k+1)} & k \geq 0 \\ 2\, g^{-k} W^{(-k-1)} & k < 0. \end{cases}$$



$$w_{n,g,\mathbf{q}}(\mathfrak{m}) = n^{\#\bigcirc} g^{\#\square} \prod_{\text{reg. faces } f} q_{\frac{\deg(f)}{2}}, \qquad b = \frac{1}{\pi}\arccos(n/2).$$

$$W^{(p)} = \sum_{\mathfrak{m} \text{ of perim } 2p} w_{n,g,\mathbf{q}}(\mathfrak{m}) \quad , \quad W_{\bullet}^{(p)} = \sum_{\substack{\mathfrak{m} \text{ of perim } 2p \\ \text{marked vertex}}} w_{n,g,\mathbf{q}}(\mathfrak{m}) \quad .$$



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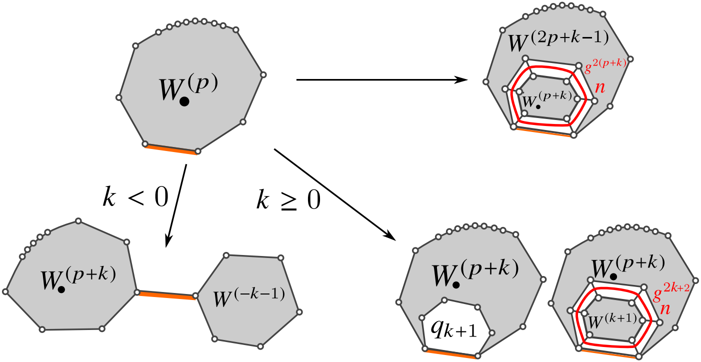

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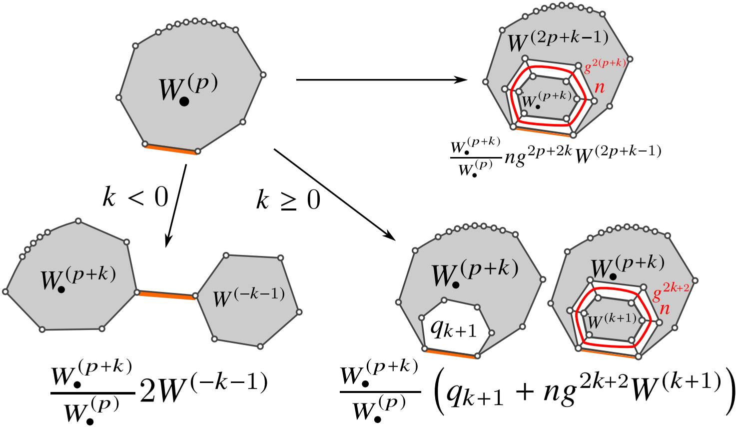

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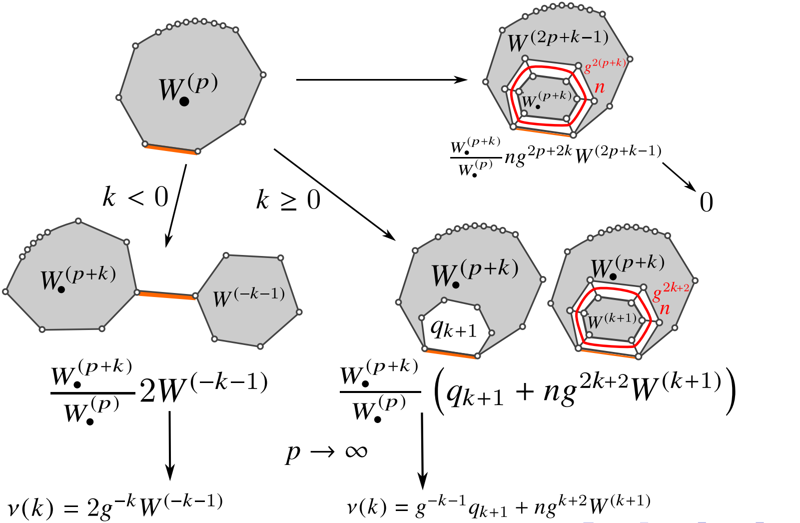

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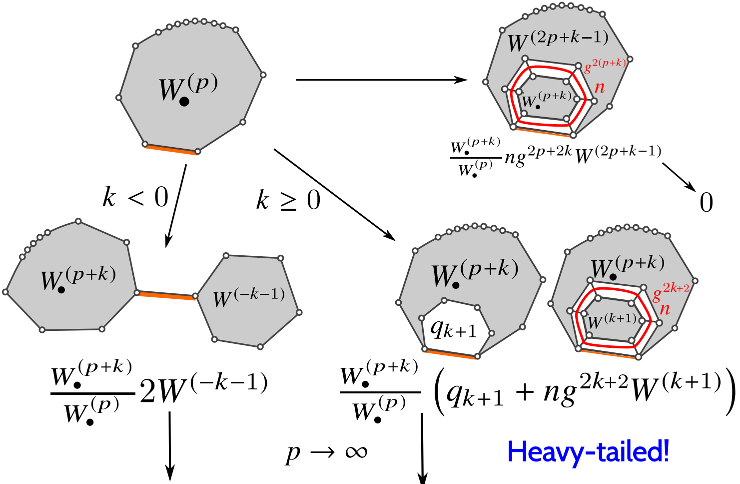
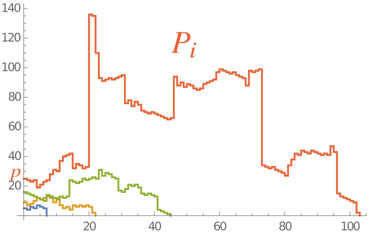
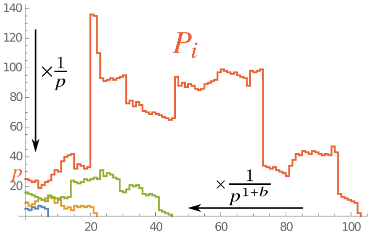
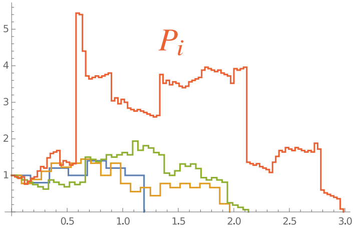


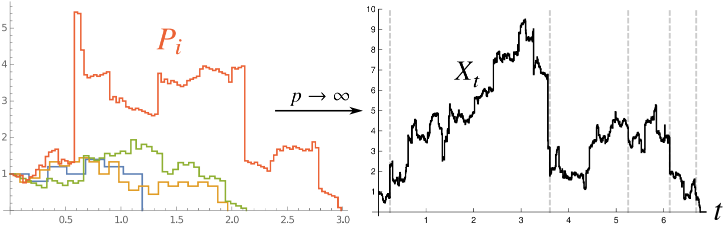
$\nu(k) = 2g^{-k} W^{(-k-1)} \sim |k|^{-2-b}$ $\qquad$ $\nu(k) = g^{-k-1} q_{k+1} + n g^{k+2} W^{(k+1)} \sim \frac{n}{2} k^{-2-b}$


P_i
p
140
120
100
80
60
40
20
20
40
60
80
100



140
120
100
80
60
40
20
p
P_i
$\times\frac{1}{p}$
$\times\frac{1}{p^{1+b}}$
20
40
60
80
100

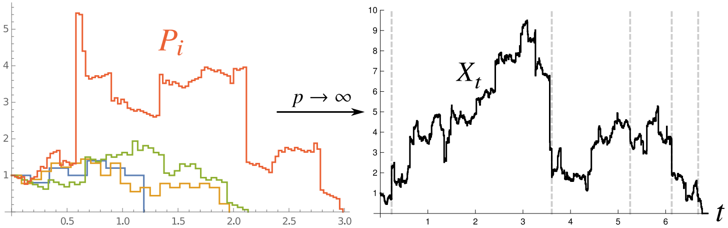

P_i
5
4
3
2
1
0.5
1.0
1.5
2.0
2.5
3.0



Theorem (TB,'18)

In the non-generic critical phase, the perimeter process $(P_i)_{i\geq 0}$ of a loop-decorated map of boundary length $2p$ satisfies the convergence

$$\left(\frac{P_{\lfloor cp^{1+b}t\rfloor}}{p}\right) \xrightarrow[p\to\infty]{(d)} (X_t)_{t\geq 0}, \qquad b = \frac{1}{\pi}\arccos\left(\frac{n}{2}\right) \in \left(0, \frac{1}{2}\right).$$



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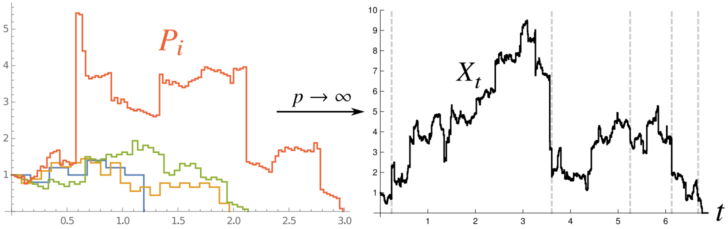
- (X_t) is a $\underbrace{\text{positive}}_{X_t>0}$ $\underbrace{\text{self-similar}}_{X_{\lambda^{1+b}t} \overset{(d)}{=} \lambda X_t}$ $\underbrace{\text{Markov process}}_{\text{memoryless}}$ (pssMp)



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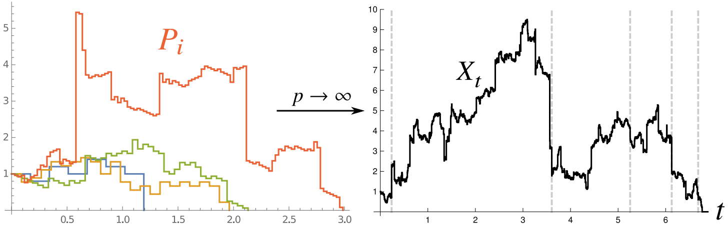
- (X_t) is a $\underbrace{\text{positive}}_{X_t > 0}$ $\underbrace{\text{self-similar}}_{X_{\lambda^{1+b}t} \overset{(d)}{=} \lambda X_t}$ $\underbrace{\text{Markov process}}_{\text{memoryless}}$ (pssMp)
- It has an explicit description as $e^{\text{Lévy process}}$, in particular

$$\mathbb{E}\left[\int_0^\infty X_t^\gamma \mathrm{d}t\right] = \frac{\pi}{\Gamma(2+3b+\gamma)\Gamma(-\gamma)(\frac{n}{2}+\cos(\pi\gamma+2\pi b))}.$$



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In the non-generic critical phase, the perimeter process $(P_i)_{i\geq 0}$ of a loop-decorated map of boundary length $2p$ satisfies the convergence

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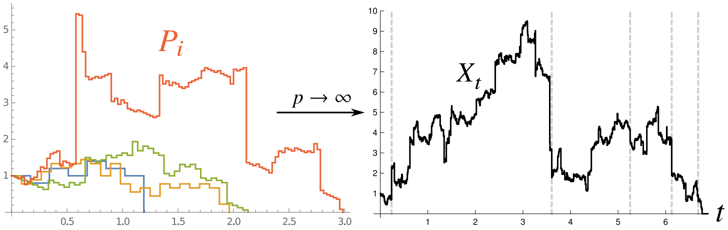

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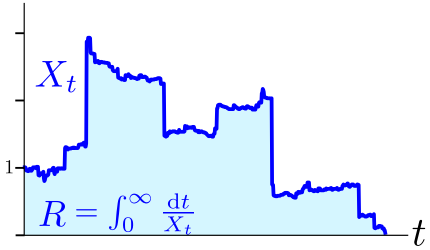

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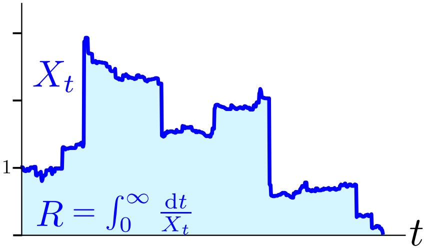

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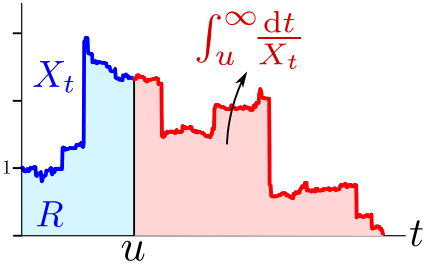

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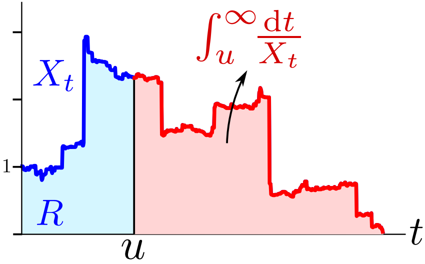

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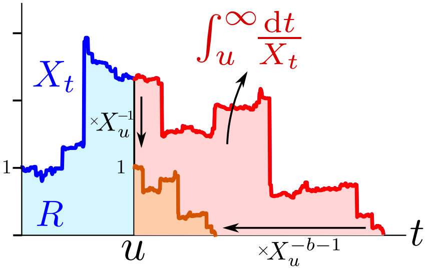

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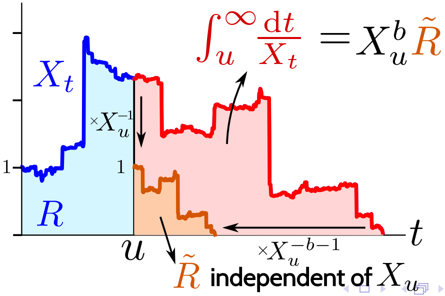

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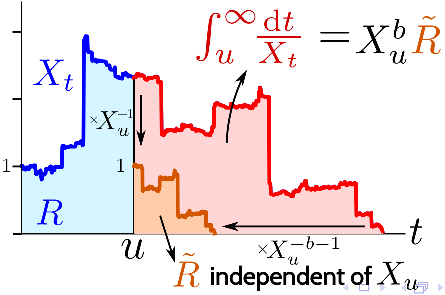

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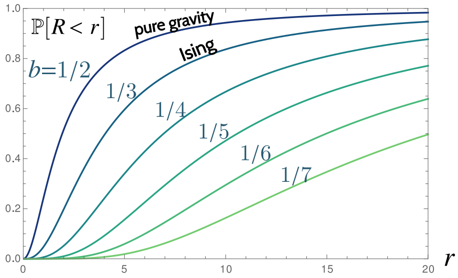

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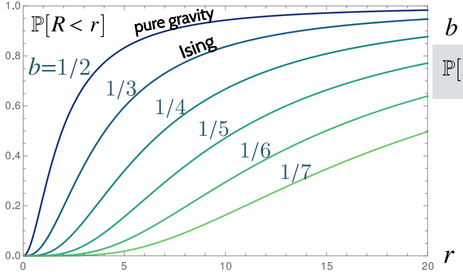

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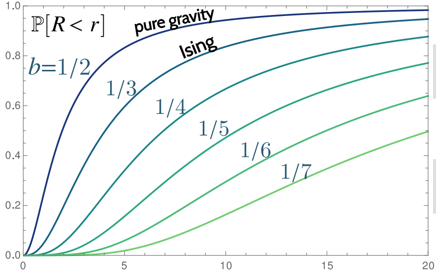

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agrees with [Bouttier, Guitter, '09]

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