Emergence of spacetime and Knitting mechanism from 2D CDT

Talk @ 2018 Nagoya International Workshop on the Physics and Mathematics of Discrete Geometries held on 6/11/2018

CDT sculpture in Nijmegen

Titech Yoshiyuki WATABIKI
(in collaboration with Jan Ambjørn)

MENU 1

1. ***Review of*** (2Dim) ***DT***
   a. What is DT **(Dynamical Triangulation)** ?
   b. DT expressed by string field theory

2. ***Review of*** (2Dim) ***CDT***
   a. What is CDT **(Causal Dynamical Triangulation)** ?
   b. CDT expressed by string field theory

3. ***DT and CDT with $\mathcal{W}$-algebra***

   [Ambjørn, Watabiki: arXiv:1505.04353]

   a. The mode expansion of DT and reduced $\mathcal{W}$-algebra
   b. The mode expansion of CDT and $\mathcal{W}$-algebra
   c. Emergence of Space(-Time & Death of the World)

MENU 2

4. **High-dimensional CDT with $\mathcal{W}$-algebra**

   [Ambjørn, Watabiki: arXiv:1703.04402]

   a. High-dimensional CDT with $\mathcal{W}$-algebra
   b. (New $\mathcal{W}$-algebra with Jordan algebra)
   c. Tangent and Hyperbolic Tangent expansions
   d. Knitting mechanism and Vanishing cosmo. const.

5. **Accelerating Universe by Fractal Strucure**

   [Ambjørn, Watabiki: arXiv:1709.06497]

   a. The modified Friedmann equation
   b. Accelerating Universe by Fractal Structure
   c. Predictions

6. **Summary and Discussions**

1. Review of DT

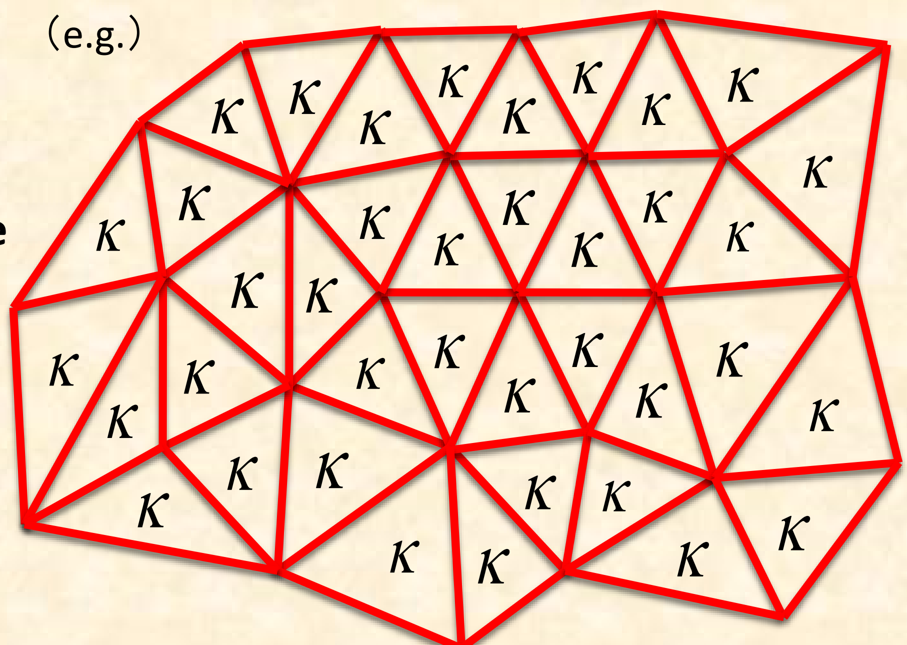
a. What is (2 dim) DT (Dynamical Triangulation) ?

- **Definition of DT**

Construction of lattice by "equilateral triangles"

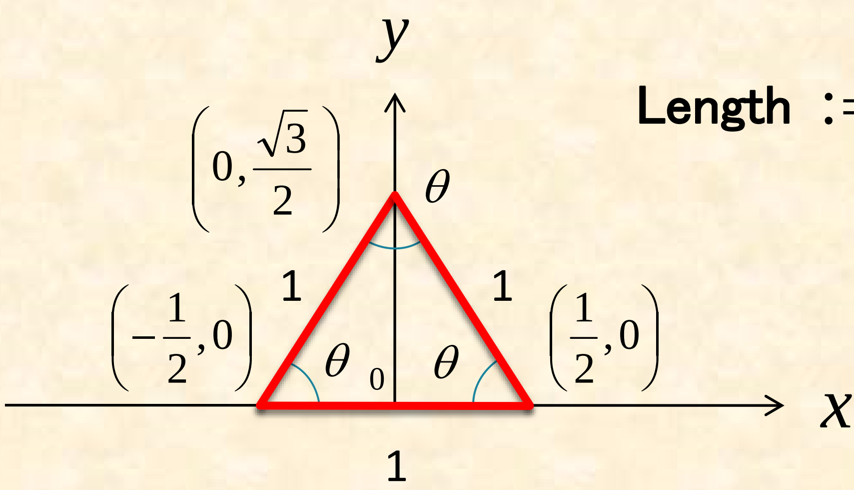
(e.g.)

All triangles are the same equilateral triangle



One triangle corresponds to one κ

Each triangle is the same size and equilateral.



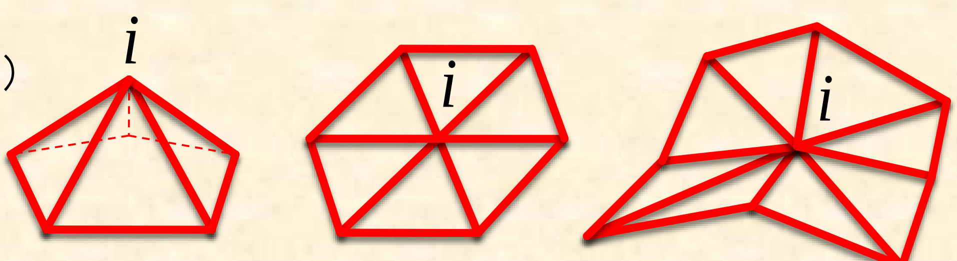
Length $:= \sqrt{x^2 + y^2}$

Curvature of site i is $\sqrt{g}R_i = (6 - q_i)\theta$

(curvature exists only on sites)

q_i is the nr. of triangles together to the site i

(e.g.)



$q_i = 5$ Curv. > 0

$q_i = 6$ Curv. = 0

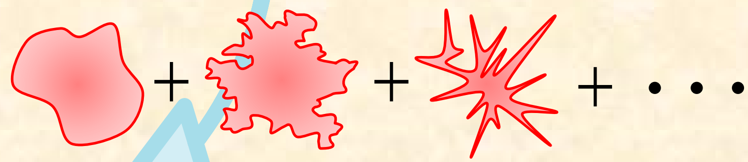
$q_i = 8$ Curv. < 0

- **Partition function of DT**

Quantum gravity is the path integral of metric $g_{\mu\nu}$
(μ is the cosmological constant)

$$Z = \int Dg_{\mu\nu}\, \mathrm{e}^{\int \mathrm{d}^2 x \sqrt{g}\,(-\frac{R}{16\pi G} + \mu)}$$

The metric $g_{\mu\nu}$ expresses various curved spaces, so the path integral is the summation of all kinds of triangulated spaces.



$$Z = \sum N^{\chi} \kappa^{N_2}$$

summation of triangulated lattices

$$\begin{cases} N = \mathrm{e}^{1/4G} \quad \kappa = \mathrm{e}^{\varepsilon^2 \mu} \\ N_2 = \dfrac{1}{\varepsilon^2} \int \mathrm{d}^2 x \sqrt{g} \end{cases}$$

ε^2 is the area of one triangle.

(κ is cosmological constant at lattice level, N_2 is nr. of triangles)

- **DT and Amplitudes** (Discrete Laplace transf.)

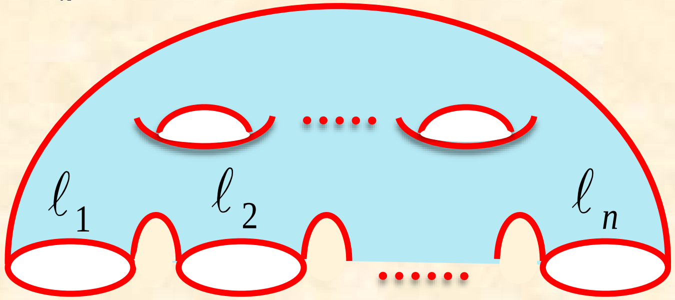
Definition of Amplitudes

The partition fun with general topology is obtained by summing up the lattice with the following topology

$$W(x_1,\ldots,x_n) := N^{n-2}\sum_{\ell_1=0}^{\infty}\cdots\sum_{\ell_n=0}^{\infty} x_1^{-\ell_1-1}\cdots x_n^{-\ell_n-1} W(\ell_1,\ldots,\ell_n)$$

W is the partition fun. fixing the topology

(Surface with n holes and several handles)



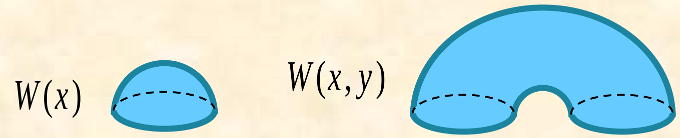
ℓ_k is the number of links at the k-th boundary.

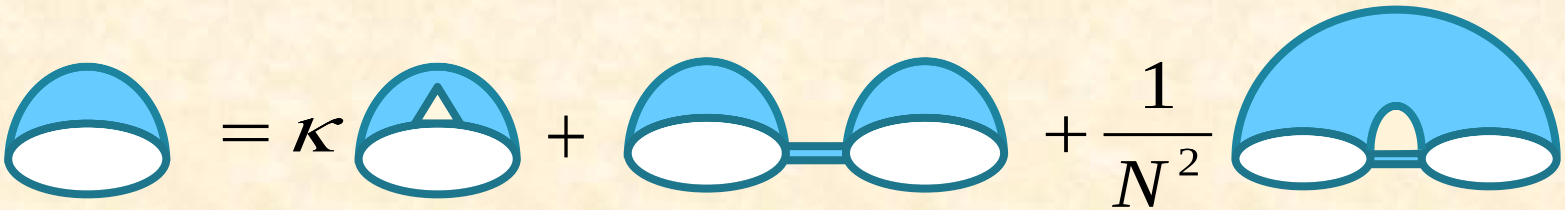
Loop equation

One obtains the loop equation for disk amplitude.

$$\int_C \frac{\mathrm{d}z}{2\pi i}\frac{V'(z)}{x-z}W(z) = W^2(x) + \frac{1}{N^2}W(x,x)$$

where $V'(z) = z - \kappa z^2$





Continuum limit of Amplitudes

The continuum limit is obtained by

$$t \to \frac{t}{\varepsilon^{1/2}} \qquad x \to x_c e^{-\varepsilon \xi} \qquad \kappa \to \kappa_c e^{\varepsilon^2 \mu}$$

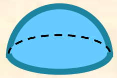
$$tH \to \frac{t}{\varepsilon^{1/2}} H = TH \qquad \left[\frac{t}{\varepsilon^{1/2}} = T \right]$$

$$x^{\ell} \to x_c^{\ell} e^{-\varepsilon \xi \ell} = x_c^{\ell} e^{-\xi L} \qquad \left[\varepsilon \ell = L \right]$$

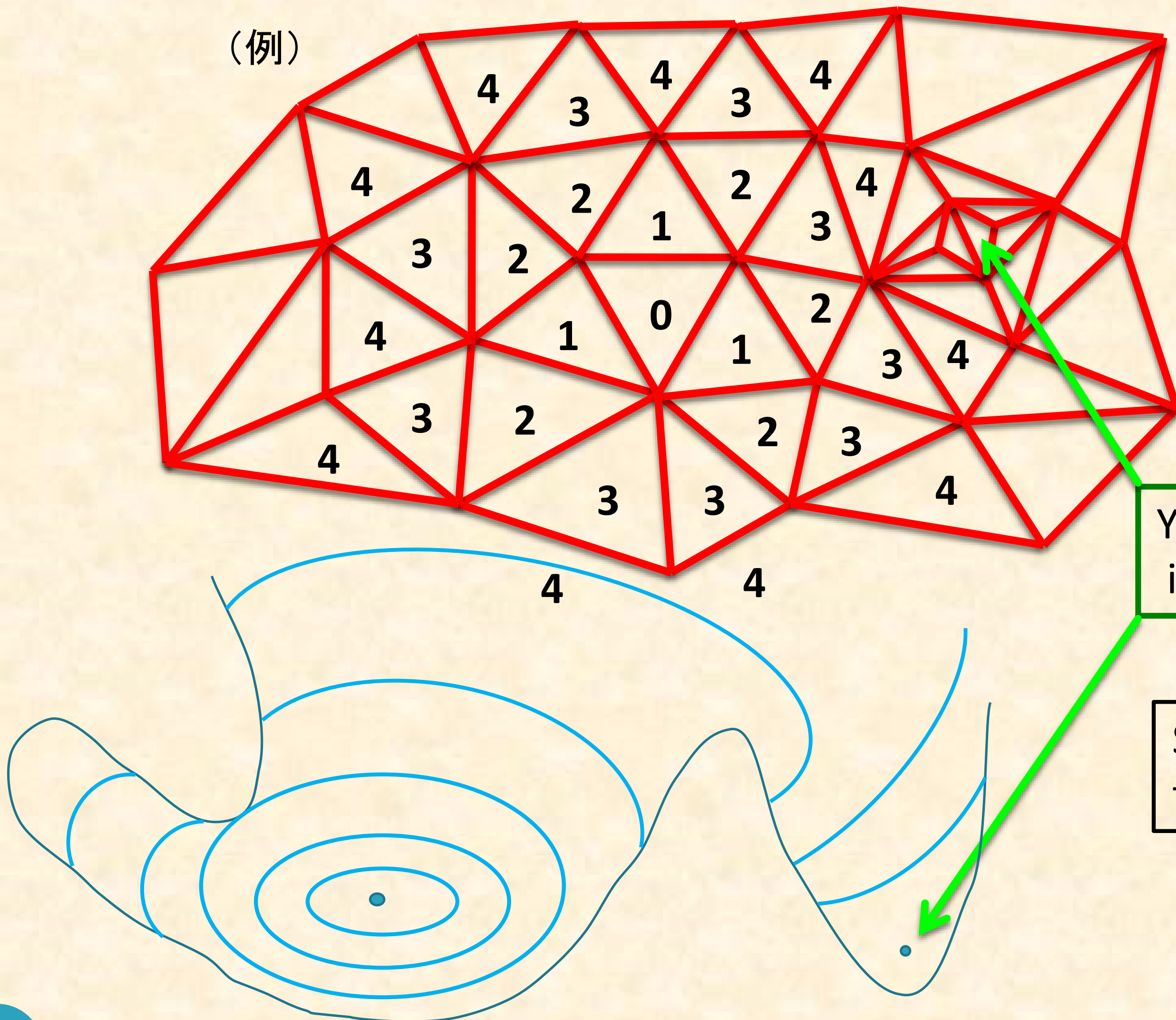
$$\kappa^{N_2} \to \kappa_c^{N_2} e^{\varepsilon^2 \mu N_2} = \kappa_c^{N_2} e^{\mu V_2} \qquad \left[\varepsilon^2 N_2 = V_2 \right]$$

The disk amplitude becomes

$$W(\xi) = \left(\xi - \frac{\sqrt{\mu}}{2} \right) \sqrt{\xi + \sqrt{\mu}}$$

$W(\xi)$ 

- **Geodesic Distance** t



You can reach this point if the surface is connected.

↓

Spaces are not created from nothing.

↓

$$H_{\mathrm{DT}} \,|\, \mathrm{phys}\rangle = 0$$

b. DT expressed by string field theory

- **Creation op. and annihilation op.** (L : length)

$$[\Psi(L),\Psi^\dagger(L')] = L\,\delta(L-L')$$

(others are zero)

- **Free Hamiltonian and Green function**

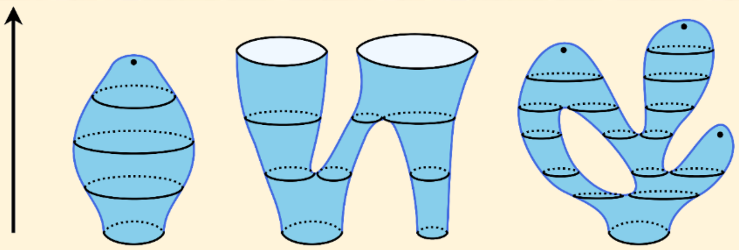
$$[\Psi(L),\Psi^\dagger(L')]^\dagger = [\Psi(L'),\Psi^\dagger(L)]$$

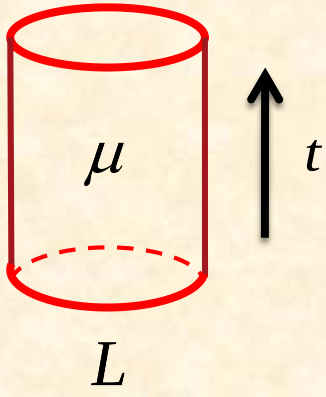
$$H_0 = 0$$

This comes from the property $\int d^2x\sqrt{g}\,R = \text{const.}$

Kawai Kawamoto
Mogami Watabiki

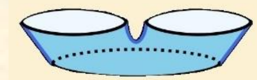
$$G(L,L',t) = \langle 0|\Psi(L')e^{-tH_0}\Psi^\dagger(L)|0\rangle$$



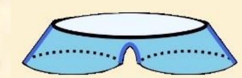


- **Hamiltonian with interactions** Ishibashi Kawai

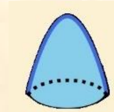
$$H_{\mathrm{DT}} = H_0 - g\int \mathrm{d}L_1 \int \mathrm{d}L_2 \Psi^{\dagger}(L_1)\Psi^{\dagger}(L_2)\Psi(L_1+L_2)$$



$$- Gg\int \mathrm{d}L_1 \int \mathrm{d}L_2 \Psi^{\dagger}(L_1+L_2)\Psi(L_1)\Psi(L_2)$$



$$- \int \frac{\mathrm{d}L}{L}\left(3\delta''(L) - \frac{3\mu}{4}\delta(L)\right)\Psi(L)$$



g **and** G **are coupling constants of string theory**

The fractal structure of 2dim. space leads to splitting and merging interactions of universes like a quantum cosmology.

- **Spaces are not created from nothing**

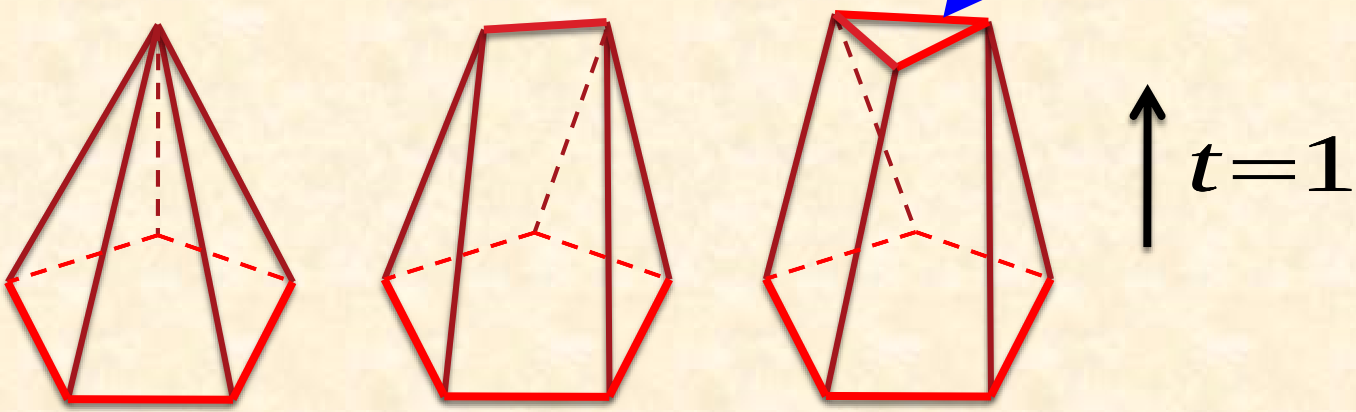
$$H_{\mathrm{DT}}\,|\,\mathrm{phys}\rangle = 0$$

This condition comes from the property of geodesic distance.

- **Tadpole term**

$$\rho(L) = 3\delta''(L) - \frac{3\mu}{4}\delta(L)$$

The dependence of μ comes from this spacetime.



The cosmological constant μ appears.

2. Review of CDT

Ambjørn Loll

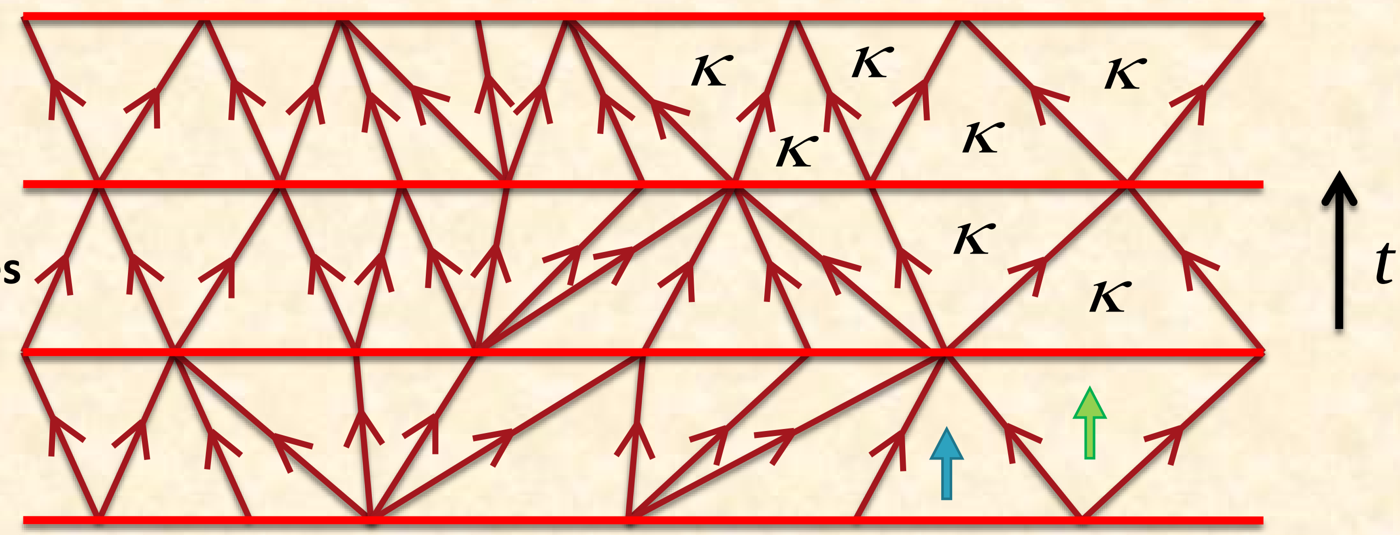
a. What is (2 dim) CDT (Causal Dynamical Triangulation) ?

- Definition of CDT

Construction of lattice by "time (isosceles) triangles"

(e.g.)

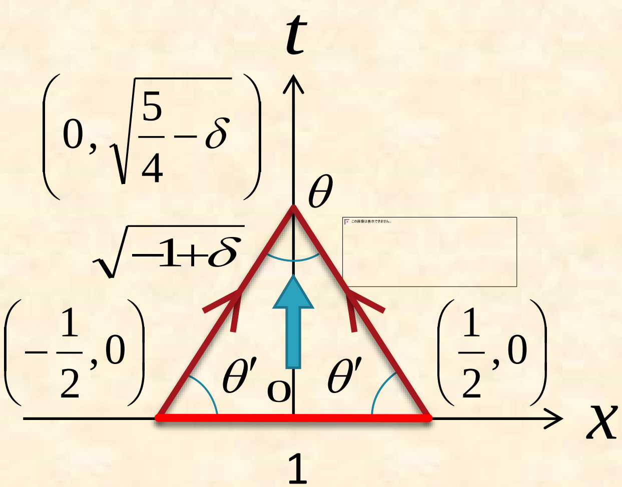
All triangles are the same isosceles triangles

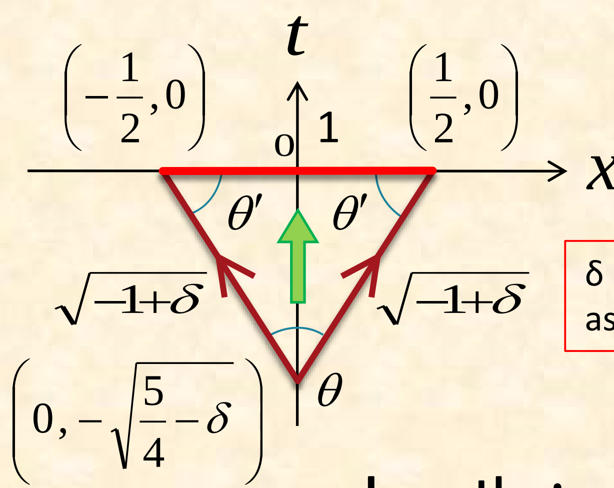


Two kinds of triangle appear.

The direction of time is unique and causal.

Each triangle is the same size and isosceles.



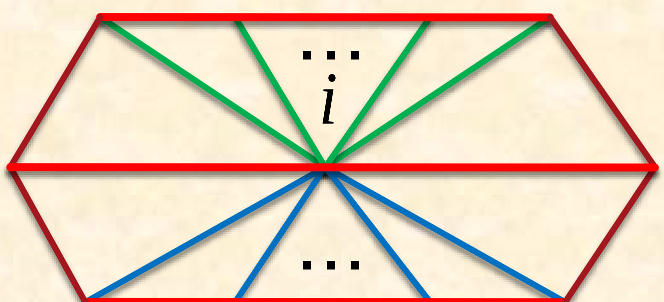


$(\delta < 1)$

δ is related with the asymmetry of *x* and *t*

Length $:= \sqrt{x^2 - t^2}$

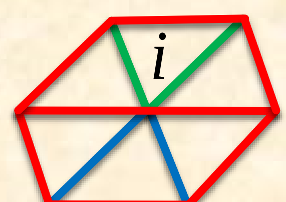
Curvature of site i **is** $\sqrt{g}R_i = (4 - k_i - j_i)\theta$



k_i Is nr. of green links

j_i Is nr. of blue links

(e.g.)



$k_i = j_i = 2$

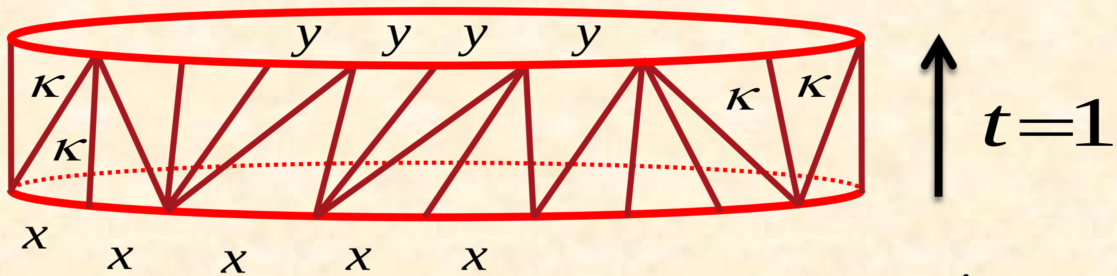
Curv. = 0

- **CDT and Green fun** (Discrete Laplace transf.)

Definition of Green fun

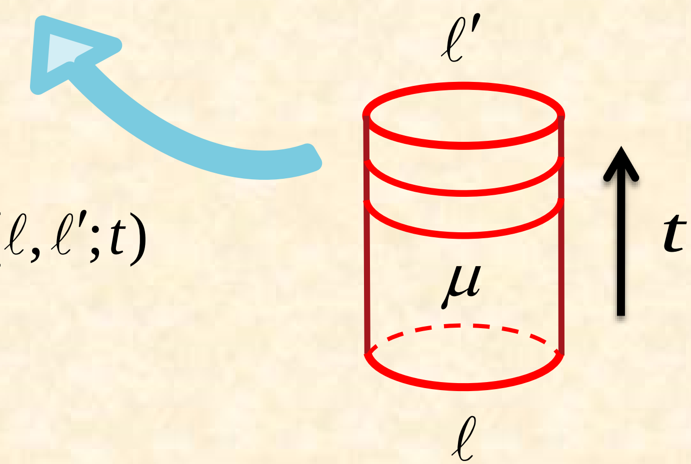
The partition fun with cylinder topology is obtained by piling the following lattice

(e.g.)



Green function is

$$G(x,y;t) = \sum_{\ell=1}^{\infty}\sum_{\ell'=1}^{\infty} x^{\ell} y^{\ell'} G(\ell,\ell';t)$$



Continuum limit of Amplitudes

The continuum limit is obtained by

$$t \to \frac{t}{\varepsilon} \qquad x \to x_{\mathrm{c}}\, \mathrm{e}^{-\varepsilon\xi} \qquad \kappa \to \kappa_{\mathrm{c}}\, \mathrm{e}^{\varepsilon^2\mu}$$

The differential equation of Green fun is

$$\frac{\partial}{\partial t} G(\xi,\eta;t) = -\frac{\partial}{\partial \xi}\Big((\xi^2-\mu)G(\xi,\eta;t)\Big)$$

$$\frac{\partial}{\partial t} G(L,L',t) = L\left(-\frac{\partial^2}{\partial L^2}+\mu\right)G(L,L',t)$$

b. CDT expressed by string field theory

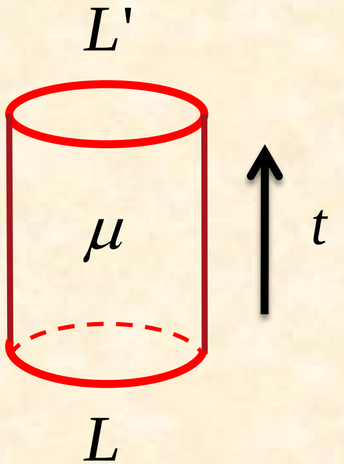
- **Creation op. and annihilation op.** (L : length)

$$[\Psi(L)\,,\Psi^\dagger(L')] = L\,\delta(L-L') \qquad \text{(others are zero)}$$

- **Free Hamiltonian and Green function**

$$H_0 = \int_0^\infty \frac{\mathrm{d}L}{L}\Psi^\dagger(L)\,L\left(-\frac{\partial^2}{\partial L^2}+\mu\right)\Psi(L)$$

(Time reversal symmetry is not broken.)



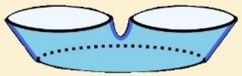
$$G(L,L',t) = \langle 0|\Psi(L')\mathrm{e}^{-tH_0}\Psi^\dagger(L)|0\rangle$$

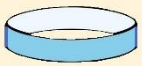
The fractal structure of CDT is weaker than that of DT, because $H_0 \neq 0$ in CDT and $H_0 = 0$ in DT. ← This reason will be discussed later.

- **Hamiltonian with interactions**

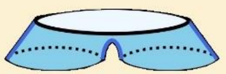
Ambjørn Loll Watabiki Westra Zohren

$$H_{\mathrm{CDT}} = H_0 - g\int \mathrm{d}L_1 \int \mathrm{d}L_2 \Psi^\dagger(L_1)\Psi^\dagger(L_2)\Psi(L_1+L_2)$$

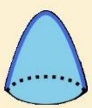




$$- Gg\int \mathrm{d}L_1 \int \mathrm{d}L_2 \Psi^\dagger(L_1+L_2)\Psi(L_1)\Psi(L_2)$$



$$- \int \frac{\mathrm{d}L}{L}\delta(L)\Psi(L)$$



The interaction terms are introduced by imitating those of DT.

g and G are coupling constants of string theory

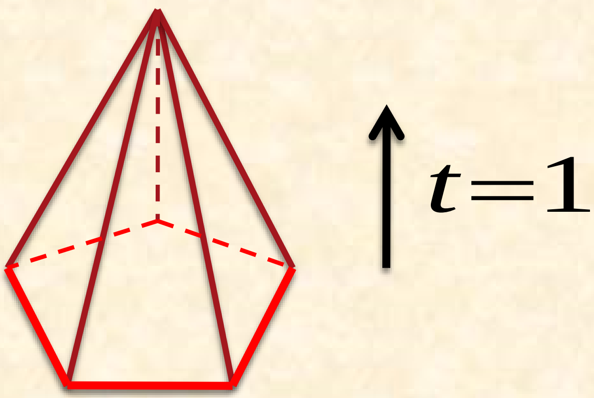
The splitting and merging interactions of universes are introduced as the same as 2dim. DT model.

- **Spaces are not created from nothing**

$$H_{\mathrm{CDT}}\,|\,\mathrm{phys}\rangle = 0$$

This is the assumption in the case of CDT.

- **Tadpole term** $\rho(L) = \delta(L)$



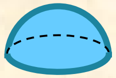
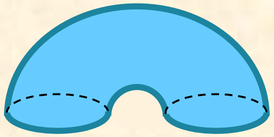
The cosmological constant μ does not appear.

- **Loop equation**

One obtains the loop equation for disk amplitude.

$$\partial_\xi \left(-V'(\xi) W(\xi) + W^2(\xi) + G g W(\xi,\xi) \right) - \frac{1}{g} = 0$$

where $V'(\xi) = \frac{1}{g}\left(\mu - \xi^2\right)$

$W(\xi)$  $W(\xi,\eta)$ 

The desk amplitude becomes

$$W(\xi) = \int_0^\infty \mathrm{d}t\, G(\xi, \ell'=0; t) = \frac{1}{\xi + \sqrt{\mu}}$$

3. DT and CDT with W-algebra

a. The mode expansion of DT and reduced W-algebra

- **Laplace transformation of the string field of DT**

$$\Psi^\dagger(\zeta) = \int_0^\infty dL\, e^{-\zeta L} \Psi^\dagger(L) \qquad \Psi(\zeta) = \int_0^\infty dL\, e^{-\zeta L} \Psi(L)$$

- **Mode expansion of the string field of DT**

$$\Psi^\dagger(\zeta) = (\text{polynomial of } \zeta) + \zeta^{3/2} - \frac{3}{8}\mu\zeta^{-1/2} + \sum_{l=1}^\infty \zeta^{-l/2-1}\phi_l^\dagger$$

$$\Psi(\zeta) = \sum_{l=1}^\infty \zeta^{l/2}\phi_l \qquad [\phi_m^\dagger, \phi_n] = m\delta_{m+n,0}$$

μ is the cosmological constant.

- **Hamiltonian and 2-reduced $\mathcal{W}$-operator**

Fukuma Kawai Nakayama

Dijkgraaf Verlinde Verlinde

Ambjørn Watabiki

$$H_{\rm DT} = -2\sqrt{G}\,\overline{W}^{(3)}_{-2} + \frac{1}{2\sqrt{G}}\phi_6^\dagger - \frac{3\mu}{8\sqrt{G}}\phi_2^\dagger$$

$$\overline{W}^{(3)}_n = \frac{1}{4}\left(\frac{1}{3}\sum_{k+l+m=2n} :\alpha_k\alpha_l\alpha_m: + \frac{1}{4}\alpha_{2n}\right)$$

$$\alpha_n = \begin{cases} n\,(\lambda_{-n} - \sqrt{G}\phi_{-n}) & (n<0) \\ 0 & (n=0) \\ \frac{1}{\sqrt{G}}\phi_n^\dagger & (n>0) \end{cases}$$

$$[\alpha_m, \alpha_n] = m\delta_{m+n,0}$$

$$\lambda_5 = \frac{1}{\sqrt{G}} \qquad \lambda_1 = -\frac{3\mu}{8\sqrt{G}}$$

b. The mode expansion of CDT and $\mathcal{W}$-algebra

- **Laplace transformation of the string field of DT**

$$\Psi^{\dagger}(\zeta)=\int_{0}^{\infty} dL\, e^{-\varsigma L}\Psi^{\dagger}(L) \qquad \Psi(\zeta)=\int_{0}^{\infty} dL\, e^{-\varsigma L}\Psi(L)$$

- **Mode expansion of the string field of DT**

$$\Psi^{\dagger}(\zeta)=(\text{polynomial of } \zeta)+\zeta^{-1}+\sum_{l=1}^{\infty}\zeta^{-l-1}\phi_{l}^{\dagger}$$

$$\Psi(\zeta)=\sum_{l=1}^{\infty}\zeta^{l}\phi_{l}$$

- **Hamiltonian and $\mathcal{W}$-operator**

$$H_{\mathrm{CDT}} = -g\sqrt{G}\,W^{(3)}_{-2} + \frac{1}{G}\left(\frac{\mu^2}{4g} + \frac{1}{4g}\phi^\dagger_4 - \frac{\mu}{2g}\phi^\dagger_2 + \phi^\dagger_1\right)$$

$$W^{(3)}_n = \frac{1}{3}\sum_{k+l+m=n} :\alpha_k \alpha_l \alpha_m:$$

$$[\alpha_m, \alpha_n] = m\delta_{m+n,0}$$

$$\alpha_n = \begin{cases} n(\lambda_{-n} - \sqrt{G}\phi_{-n}) & (n<0) \\ \nu & (n=0) \\ \dfrac{1}{\sqrt{G}}\phi^\dagger_n & (n>0) \end{cases}$$

$$\lambda_3 = \frac{1}{6g\sqrt{G}} \qquad \lambda_1 = -\frac{\mu}{2g\sqrt{G}}$$

$$\nu = \frac{1}{\sqrt{G}}$$

- **No. of Parameters**

DT models: There are 3 parameters, g, G, μ.

But, g can be removed by scaling t as

(∗) $$t \to \frac{t}{\sqrt{g}} \qquad \Psi^\dagger \to \frac{\Psi^\dagger}{\sqrt{g}} \qquad \Psi \to \sqrt{g}\,\Psi \qquad G \to \frac{G}{g}$$

CDT models: There are 3 parameters, g, G, μ.

No such scaling like (∗) exists

because of the non-vanishing kinetic term.

c. Emergence of Space(-Time & Death of the World)

- **DT case**

$$H_{\mathrm{DT}}\,|\,\mathrm{phys}\rangle = 0$$

$$\overline{H}_W\,|\,\mathrm{phys}\rangle = -\frac{1}{\sqrt{G}}\left(\frac{1}{2}\phi_6^\dagger - \frac{3\mu}{8}\phi_2^\dagger\right)|\,\mathrm{phys}\rangle \neq 0$$

$$\overline{H}_W := -2\sqrt{G}\,\overline{W}_{-2}^{(3)}$$

$$\lim_{t\to\infty}\langle 0|\,e^{-tH}\,\phi_{2n}^\dagger\,|\,\mathrm{phys}\rangle = 0$$ (Disk amplitude with boundary $2n$)

⟶ There is no essential difference between H_{DT} and $\overline{H}_W$

- **CDT case**

$$H_{\mathrm{CDT}} \,|\, \mathrm{phys}\rangle = 0$$

$$H_W \,|\, \mathrm{phys}\rangle = -\frac{1}{G}\left(\frac{\mu^2}{4g} + \frac{1}{4g}\phi_4^\dagger - \frac{\mu}{2g}\phi_2^\dagger + \phi_1^\dagger\right)|\, \mathrm{phys}\rangle \neq 0$$

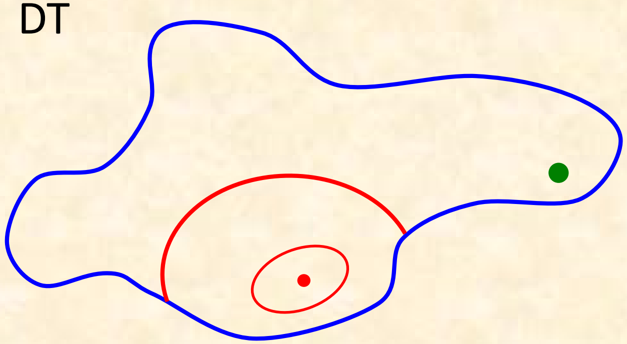
$$H_W := -g\sqrt{G}\, W_{-2}^{(3)}$$

⟶ There exists the difference between H_{CDT} and H_W

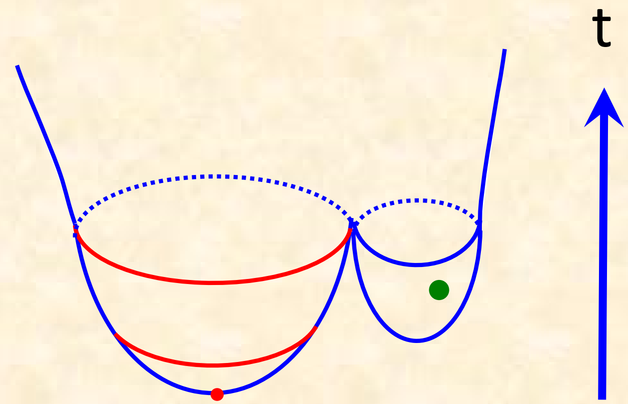
- **The vacuum condition and The geometrical condition**

You can (not) reach ● from ●

DT



CDT



$$H_{\mathrm{DT}}\,|\,\mathrm{phys}\rangle = 0$$

This condition can **always** be applied to any configurations because of geometric property.

$$H_{\mathrm{CDT}}\,|\,\mathrm{phys}\rangle = 0$$

This condition can **not** be applied to any configurations because of geometric property.

→ There is no reason to stick to this condition.

- **Emergence of Space**

From now on, we assume the following Hamiltonian

$$H_W = \mu\phi_1 - 2g\phi_2 - gG\phi_1\phi_1 - \frac{1}{G}\left(\frac{\mu^2}{4g} + \frac{1}{4g}\phi_4^\dagger - \frac{\mu}{2g}\phi_2^\dagger + \phi_1^\dagger\right)$$

$$- \sum_{l=1}^{\infty}\phi_{l+1}^\dagger l\phi_l + \mu\sum_{l=2}^{\infty}\phi_{l-1}^\dagger l\phi_l - 2g\sum_{l=3}^{\infty}\phi_{l-2}^\dagger l\phi_l$$

$$- g\sum_{l=4}^{\infty}\sum_{n=1}^{l-3}\phi_n^\dagger\phi_{l-n-2}^\dagger l\phi_l - gG\sum_{l=1}^{\infty}\sum_{m=\max(3-l,1)}^{\infty}\phi_{m+l-2}^\dagger m\phi_m l\phi_l$$

$$= -g\sqrt{G}\,W_{-2}^{(3)}$$

(red box) Is added from the viewpoint of $\mathcal{W}$-symmetry.

- **Emergence of Time & Death of the World**

The time and physical vacuum were born by the interaction between different Hilbert spaces of $\mathcal{W}$−algebra.

(e.g.)

$$|\lambda_3,\lambda_1,\nu\rangle \Leftrightarrow |\lambda_3',\lambda_1',\nu'\rangle \otimes |\lambda_3'',\lambda_1'',\nu''\rangle$$

The factor in front of $W_{-2}^{(3)}$ can be removed

by the redefinition of $W_{-2}^{(3)} \to \frac{1}{k^2} W_{-2}^{(3)}$ for $\alpha_n \to k^n \alpha_n$

So, the time T can be removed if all λ_n are zero.

If changing the values of λ_n is possible,
the time is born and the world dies by this process.

4. High-dim. CDT with $\mathcal{W}$-algebra

a. High−dimensional CDT with $\mathcal{W}$−algebra

- **The Hamiltonian**

$$H_W = -g\sqrt{G}\,W_{-2}^{(3)}$$

$$W_n^{(3)} := \frac{1}{3}\sum_{a,b,c} d_{abc} \sum_{k+l+m=n} :\alpha_k^{(a)}\alpha_l^{(b)}\alpha_m^{(c)}:$$

$$[T_a, T_b] = \sum_c c_{abc} T_c \qquad \{T_a, T_b\} = \frac{4}{3}\delta_{ab} + \sum_c d_{abc} T_c$$

T_a is orthogonal real, complex Hermitian,
quaternion Hermitian, or, octonion Hermitian matrix.

- In the case of 3×3 complex Hermitian matrix

$T_a\ [a=1,2,\ldots,8]$ are Gell-mann matrices.

$$d_{000}=\sqrt{\frac{2}{3}} \qquad d_{011}=\cdots=d_{088}=\sqrt{\frac{2}{3}}$$

$$d_{118}=d_{228}=d_{338}=\frac{1}{\sqrt{3}} \qquad d_{888}=-\frac{1}{\sqrt{3}}$$

$$d_{448}=d_{558}=d_{668}=d_{778}=-\frac{1}{2\sqrt{3}}$$

$$d_{344}=d_{355}=\frac{1}{2} \qquad d_{366}=d_{377}=-\frac{1}{2}$$

$$d_{146}=d_{157}=d_{256}=\frac{1}{2} \qquad d_{247}=-\frac{1}{2}$$

0th, 8th, 3rd spaces play a special role!

c. Tangent and Hyperbolic Tangent expansion (THT-expansion)

- **Physical vacuum** (we choose the following physical vacuum)

(e.g.)

$$\alpha_{-3}^{(0)} | \mathrm{phys}\rangle = \lambda_3^{(0)} | \mathrm{phys}\rangle = \frac{1}{6g\sqrt{G}} | \mathrm{phys}\rangle$$

$$\alpha_{-1}^{(0)} | \mathrm{phys}\rangle = \lambda_1^{(0)} | \mathrm{phys}\rangle = -\frac{\mu_0}{2g\sqrt{G}} | \mathrm{phys}\rangle$$

$$\alpha_{-1}^{(3)} | \mathrm{phys}\rangle = \lambda_1^{(3)} | \mathrm{phys}\rangle = -\frac{\mu'}{2g\sqrt{G}} | \mathrm{phys}\rangle$$

- **Hamiltonian which gives 3, 4, 6, 10-dim model**

$$
\begin{aligned}
H_W = &-\sum_{l=1}^{\infty}\phi_{l+1}^{(i)\dagger}\, l\,\phi_l^{(i)} - \sum_{l=1}^{\infty}\phi_{l+1}^{(I)\dagger}\, l\,\phi_l^{(I)} - \sum_{l=1}^{\infty}\phi_{l+1}^{(\bar{I})\dagger}\, l\,\phi_l^{(\bar{I})} \\
&-\sum_{l=1}^{\infty}\phi_{l+1}^{(3')\dagger}\, l\,\phi_l^{(3')} - \sum_{l=1}^{\infty}\phi_{l+1}^{(0')\dagger}\, l\,\phi_l^{(0')} \\
&+\mu_0\sum_{l=2}^{\infty}\phi_{l-1}^{(i)\dagger}\, l\,\phi_l^{(i)} + \mu_-\sum_{l=2}^{\infty}\phi_{l-1}^{(I)\dagger}\, l\,\phi_l^{(I)} - \mu_+\sum_{l=2}^{\infty}\phi_{l-1}^{(\bar{I})\dagger}\, l\,\phi_l^{(\bar{I})} \\
&+\mu_0\sum_{l=2}^{\infty}\phi_{l-1}^{(0')\dagger}\, l\,\phi_l^{(0')} + \mu'_-\sum_{l=2}^{\infty}\phi_{l-1}^{(8')\dagger}\, l\,\phi_l^{(8')} - \mu'_+\sum_{l=2}^{\infty}\phi_{l-1}^{(3')\dagger}\, l\,\phi_l^{(3')}
\end{aligned}
$$

$i = 1, 2$

$I = 4, 5$

$\bar{I} = 6, 7$

(2, 4, 8, 16) +2 spaces are expanding and form a compact spaces.

2, 3, 5, 9 spaces are expanding toward infinity.

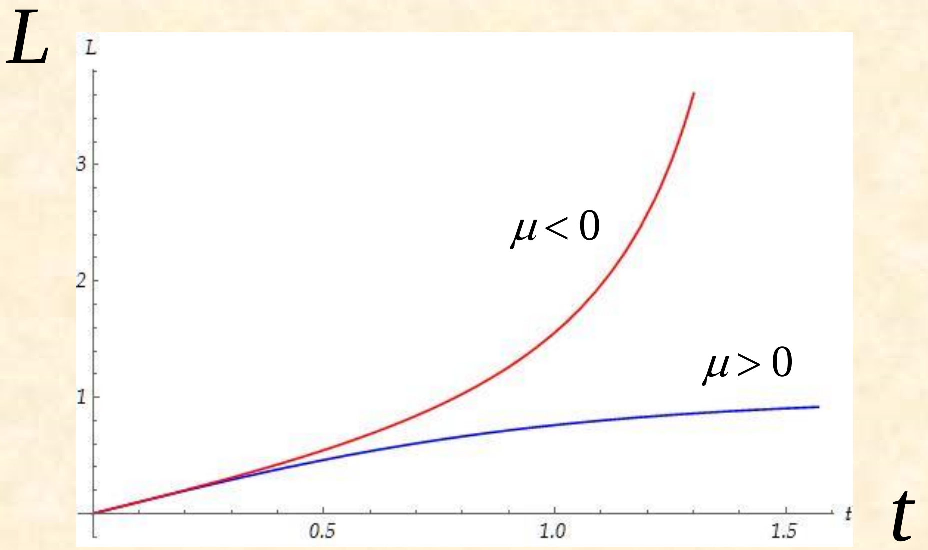
$\phi_l^{(0')\dagger}$ $\phi_l^{(8')\dagger}$ $\phi_l^{(3')\dagger}$ **are linear combination of** $\phi_l^{(0)\dagger}$ $\phi_l^{(8)\dagger}$ $\phi_l^{(3)\dagger}$

μ_+ μ_+ μ'_+ μ'_+ **are linear combination of** μ_0 μ'

- **Tangent and Hyperbolic Tangent expansion (THT-expansion)**

The length of space is growing as

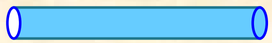
$$L(t) = \frac{\int_0^\infty \frac{dL}{L} L\, G(0,L;t)}{\int_0^\infty \frac{dL}{L} G(0,L;t)} = \begin{cases} \frac{1}{\sqrt{\mu}} \tanh \sqrt{\mu}\, t & (\mu > 0) \\ \frac{1}{\sqrt{-\mu}} \tan \sqrt{-\mu}\, t & (\mu < 0) \end{cases}$$



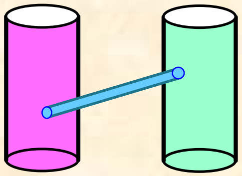
d. Knitting Mechanism and Vanishing Cosmo. Const.

- **How to knit spaces**

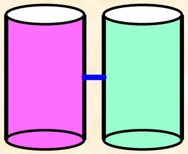
1. Prepare wormhole spaces (flavor "0") whose radii do not expand bigger than Planck length.



2. Let wormhole spaces interact spaces with flavors "a" [$a = 0, 1, \ldots, D$].



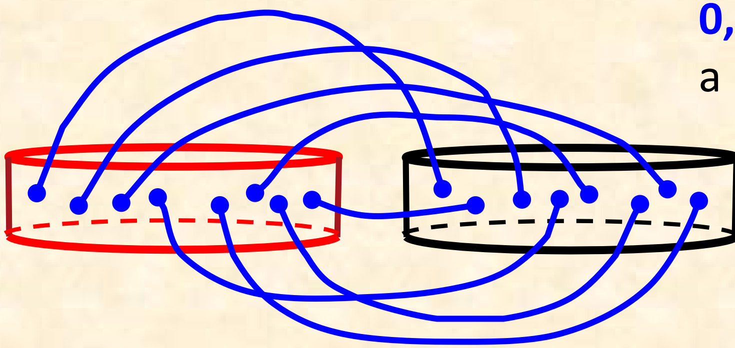
3. Let the wormhole spaces with infinitesimal radius and length be dominant.



4. Every coordinates $(x_1, \ldots, x_D)$ are connected by wormholes.

→ The set of wormholes forms a *D*-dimensional space.

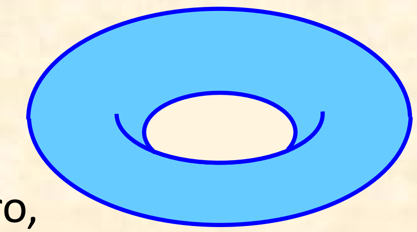
- **Knitting Mechanism** (Dimension Enhancement)

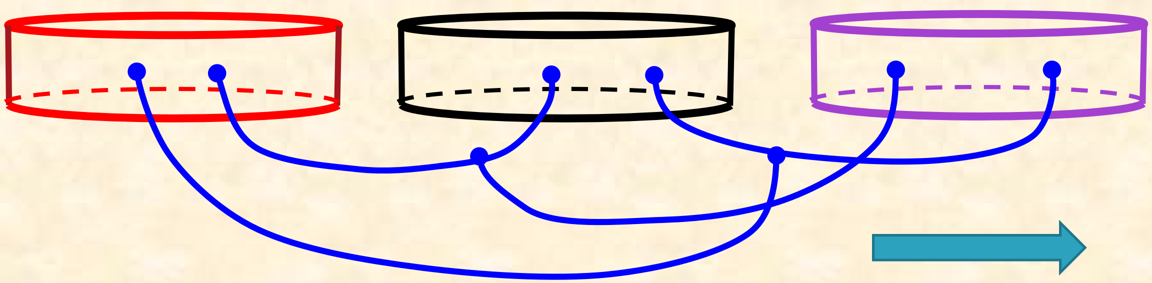


0, 8, 3th space play a role of wormholes.

The set of wormholes gives toroidal space.

If the lengths of wormholes are zero,





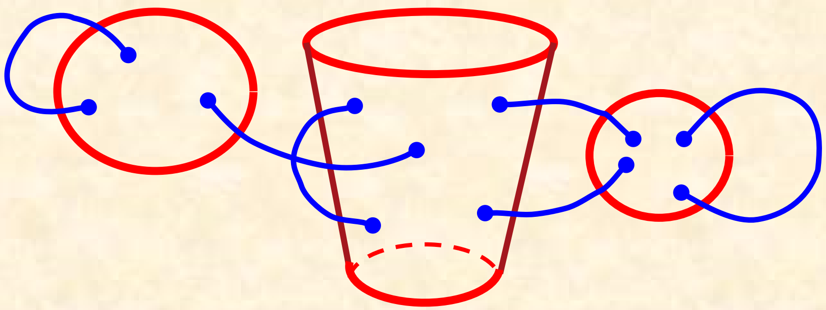
Higher dimensional toroidal space.

These wormholes are different from the standard wormholes **(Einstein–Rosen bridges)**.

- **Matter fields change to those in high dimensions.**

We assume gravitons, gauge fields, and so on appear as matter fields.

- **Vanishing cosmological constant** (Coleman mechanism)



0, 8, 3th space play a role of wormholes.

$$\mu \phi_{l-1}^{(a)\dagger} l \phi_l^{(a)} \to 0$$

Summing up all possible wormholes, cosmological term disappear.

This wormholes are different from the standard wormholes, i.e. Einstein-Rosen Bridges, because 0, 8, 3^{th} space is the different dimension.

[The curvature R is necessary to be bounded below.]

5. Accelerating Univ. by Fractal Structure

a. The modified Friedmann equation

- **The Hamiltonian from** $-\phi_{l+1}^{\dagger} l \phi_l + \mu \phi_{l-1}^{\dagger} l \phi_l - 2g\, \phi_{l-2}^{\dagger} l \phi_l$

$$\mathcal{H} = -NL\left(\Pi^2 - \mu + \frac{2g}{\Pi}\right)$$

$$\{L, \Pi\} = 1$$

This term comes from the interaction with other (baby) universe.

$N(t)$ is introduced to realize the reparametrization invariance of time.

Then, we obtain

$$\left(\frac{\dot{L}}{2NL}\right)^2 = \mu - \frac{2gNL}{\dot{L}} \frac{1+3F(x)}{(F(x))^2} \qquad x := -\frac{8gL^3}{\dot{L}^3}$$

where $F(x)$ satisfies $(F(x))^3 - (F(x))^2 + x = 0$

- **Assumption after the Big Bang**

g comes from the baby universe production.
μ disappears because of the Coleman mechanism.
On the other hand, g should survives because
it plays the coupling constant of wormholes.

In CDT, matter fields are considered to be integrated out as the same as in DT. So, we assume matter fields appear effectively after the Big Bang.

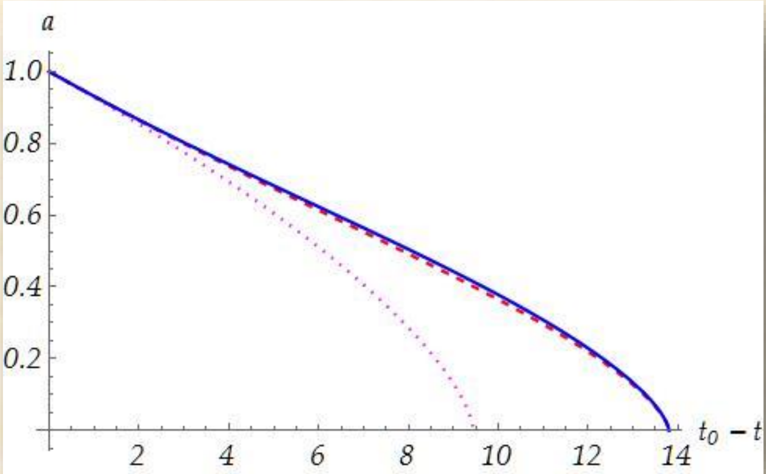
Assumption: $$\mu \to \frac{\kappa\rho}{12}$$

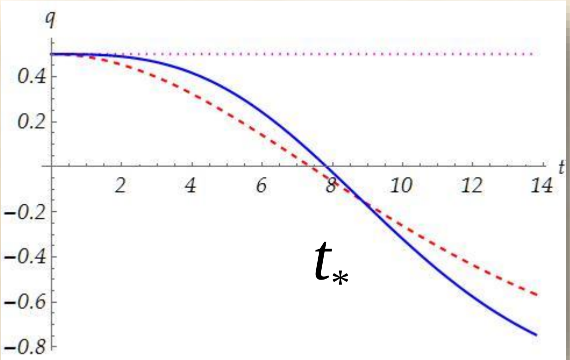
ρ is the energy density of matter
If space dimension is 3, we have $\kappa\rho = \frac{(\text{const.})}{L^3}$

b. Accelerating Universe

- **The expansion of Universe**

Solid line is our model
Dashed line is Λ-CDM
Dotted line is CDM





$$a(t) := \frac{L(t)}{L(t_0)} \qquad \left(z(t) + 1 = \frac{1}{a(t)} \right) \qquad q(t) := -\frac{\ddot{L}(t)}{(H(t))^2 L(t)}$$

The acceleration of universe is caused by the production of micro-spaces, which originates from the fractal structure of spacetime by quantum effect. Negative pressure doesn't exist because the dark energy is unnecessary.

C. Predictions

- We here assume

$$H(t_0) = 69\ [\mathrm{km\ s^{-1} Mpc^{-1}}] \qquad t_0 = 13.8\ [\mathrm{Gyr}]$$

- We predict

$$\Omega_{\mathrm{m}}(t_0) \approx 0.33 \qquad w(t_0) \approx -1.2 \qquad q(t_0) \approx -0.74$$

$$\left[\Omega_{\mathrm{m}}(t) := \frac{\kappa \rho(t)}{3(H(t))^2}\right] \qquad \left[w(t) := \frac{p(t)}{\rho(t)} = \frac{2q(t)-1}{3(1-\Omega_{\mathrm{m}}(t))}\right]$$

- Observed value

$$w(t_0) \approx -1.16\ \pm 0.19$$ (Planck + WMAP + BAO + SNIa)

Cf. $w(t) = -1$ (Λ-CDM)

DISCUSSIONS

- **Difference between THT−expansion and Inflation**

	THT-expansion	Inflation
Horizon problem	✔	✔
Flatness problem	✔ (K=0)	✔
Inflaton	none	necessary
Ending	(Coleman mechanism)	(drop off from slow-roll potential)
Eternal scenario	impossible	possible
(The eternal inflation doesn't occur because Planck scale is fixed.)		

Inflaton is unnecessary because our cosmological constant comes from the production of baby universes.

VERY OPTIMISTIC CONJECTURE (VOC)

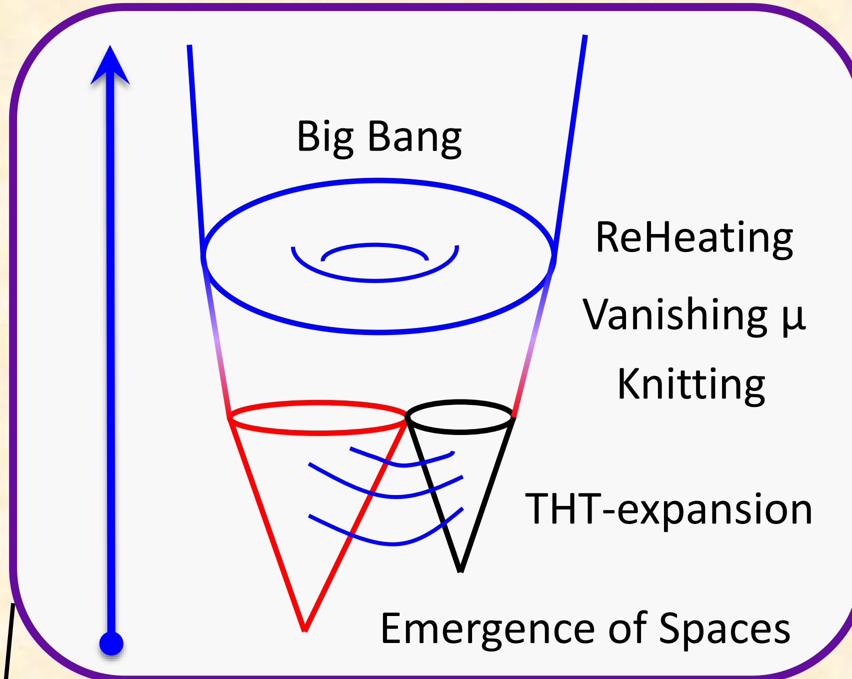
- **Real, Complex, Quaternion, Octonion $\mathcal{W}$-algebra models generates 3, 4, 6, 10-dimensional light-cone superstring theories, respectively, as matter fields.**
- **Only Octonion $\mathcal{W}$-algebra model has the Lorentz symmetry because 10-dimensional light-cone superstring theory has it.**
- **The reason why the string theories are expected**

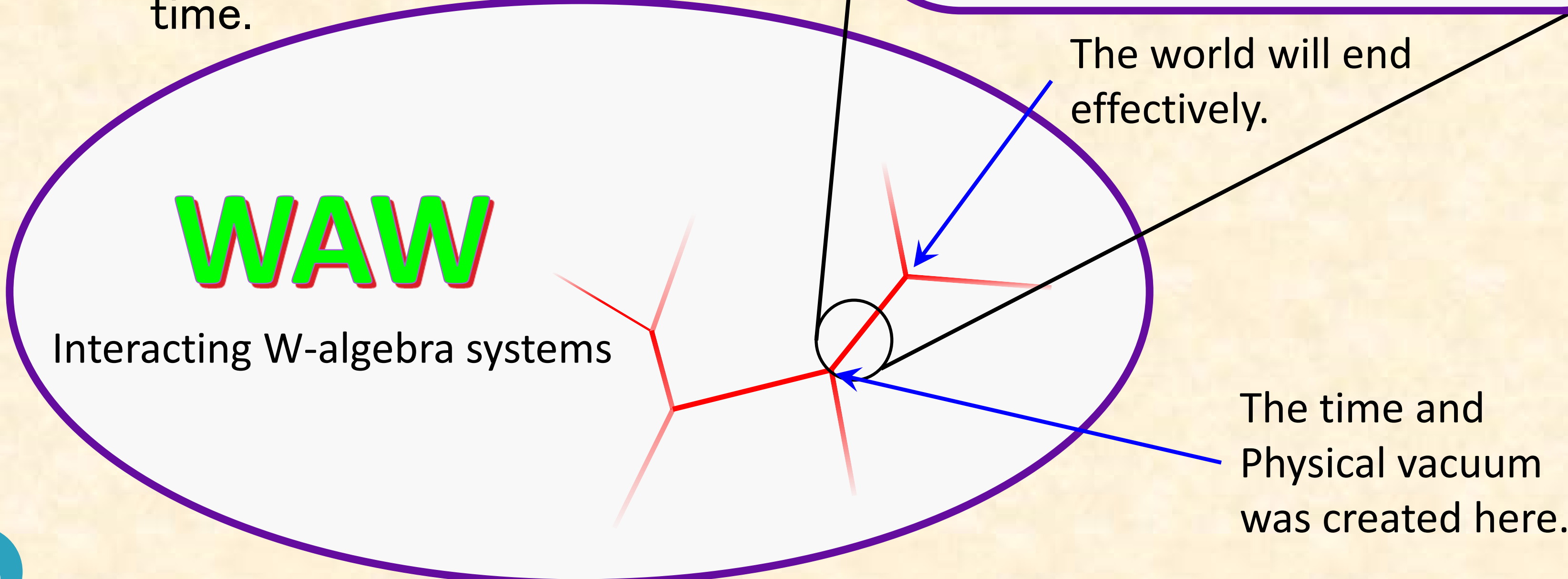
→
  - The dimensions of our model coincide with the critical dimensions of string theories.
  - Our model is the high dimensional theory of 2D quantum gravity.

SUMMARY

- **From WAW (W-alg. world) to Big Bang**

The $\mathcal{W}$−algebra world (WAW) is described by the static picture or the picture using fictitious time.





SUMMARY

- We constructed **the quantum gravity theory** from CDT.
- **The simplest model has 2, 3, 4, 6, 10 dimension spacetime.**

WAW

The time and the physical vacuum was born

The many spaces were born and THT-expansion started.

When the size of spaces exceeded Planck scale,
dimension enhancement occurred by Knitting mechanism.

By Coleman mechanism the cosmological constant vanished
and the space was **re-**heated. And then, the Big Bang started.

(The standard Big Bang scenario began)

Symmetries were increasing.

t

- **We also calculated physical parameters about the late expansion.**

PROBLEMS

- How does the knitting and Coleman mechanism occur?
- What kind of matters (graviton, gauge fields, Higgs bosons, quarks and leptons) appear?
- How is the Lorentz symmetry realized?
- Why is the parameter g so small, and is the total energy in our universe so big, compared to the Planck scale?
- What is the mathematical structure of $\mathcal{W}$-algebra?