2D forest fire processes near criticality and percolation with heavy-tailed impurities

Pierre Nolin[1] (CityU Hong Kong)

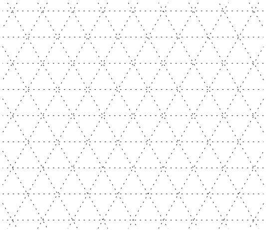
based on j.w. with Rob van den Berg (CWI and VU, Amsterdam)

November 9th, 2018

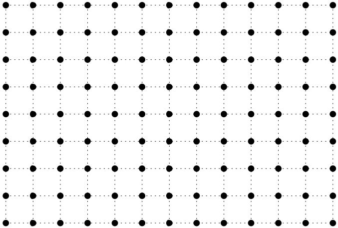
[1]partially supported by GRF grant CityU11304718 (Research Grants Council of Hong Kong SAR)

Bernoulli percolation

Model for random media: Bernoulli percolation (1957)



Site percolation on $\mathbb{T}$



Bond percolation on $\mathbb{Z}^2$

Vertices ("sites") are

- occupied / black (p)
- vacant / white ($1 - p$)

Edges ("bonds") are

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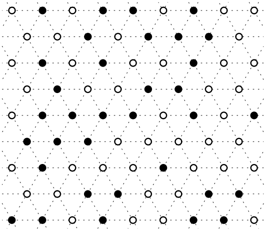

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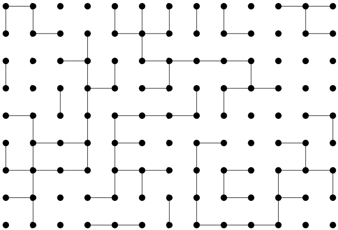

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$\rightsquigarrow$ connectivity properties?

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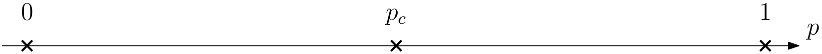
Percolation: phase transition as p varies



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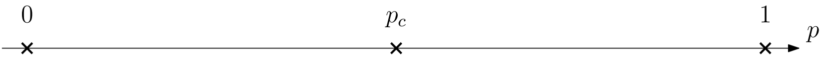
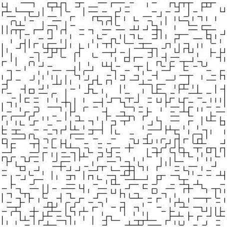


subcritical regime

no ∞ cluster

exponential decay
for cluster size

trivial large scale behavior



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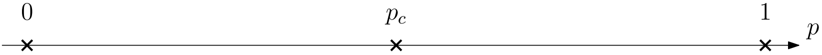
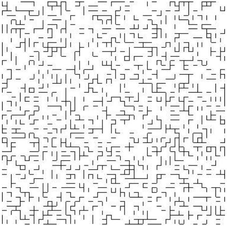
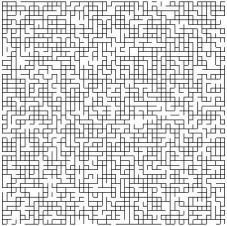


subcritical regime	*supercritical regime*
no ∞ cluster	unique ∞ cluster
exponential decay for cluster size	exponential decay for finite clusters
trivial large scale behavior	**trivial large scale behavior**





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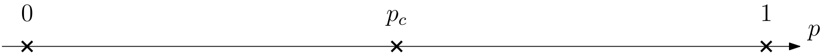

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critical regime

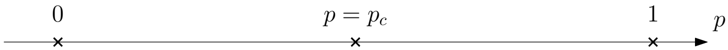
conformal invariance

non-trivial scaling limits

connection with SLE(6)

...

Near-critical regime



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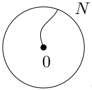
e.g. $\mathbb{P}_{p_c}\Big($ 

$\Big) = N^{-5/48+o(1)}$ $(N \to \infty)$

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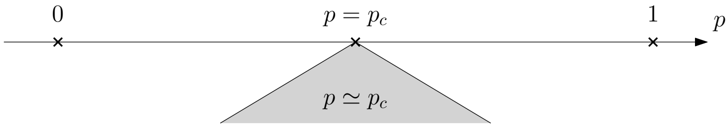

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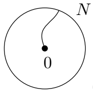
e.g. $\mathbb{P}_{p_c}\Big($ 

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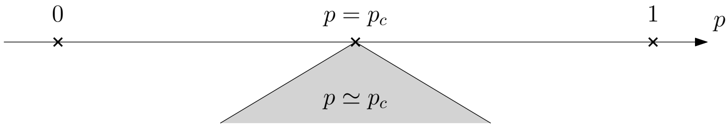

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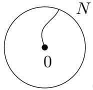
e.g. $\mathbb{P}_{p_c}\Big($ 

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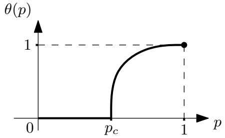

near-critical regime ($p \simeq p_c$)

e.g. density $\theta(p) = (p - p_c)^{5/36+o(1)}$ $(p \searrow p_c)$

Forest fire processes

We consider processes on a 2D lattice ($\mathbb{Z}^2$ or $\mathbb{T}$), constructed from 2 Poisson point processes: on each vertex, **births** (rate 1) and **ignitions** (rate $\zeta > 0$, typically very small)

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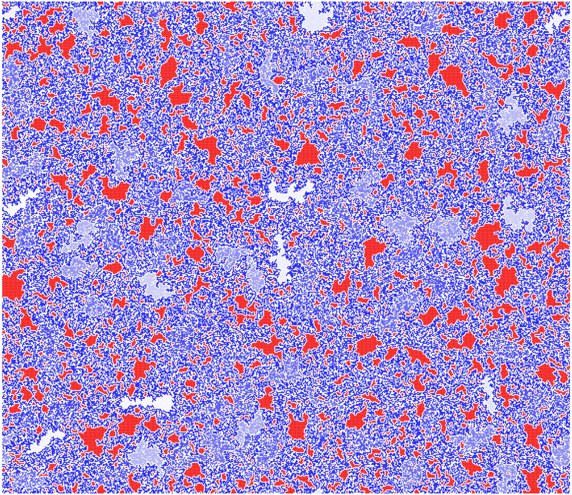

$N = 200$-volume-frozen percolation on $\mathbb{T}$ (Fig. Demeter Kiss)

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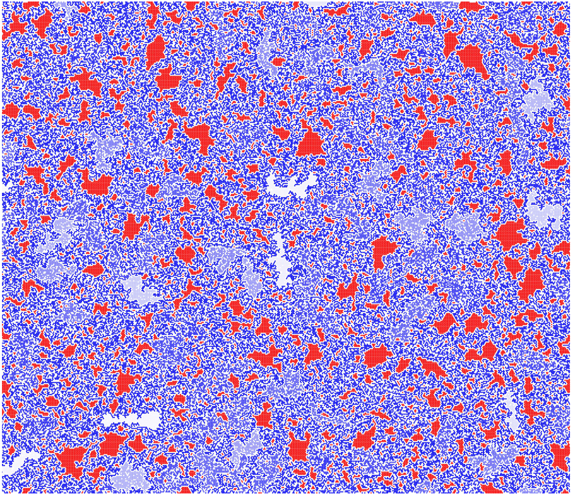

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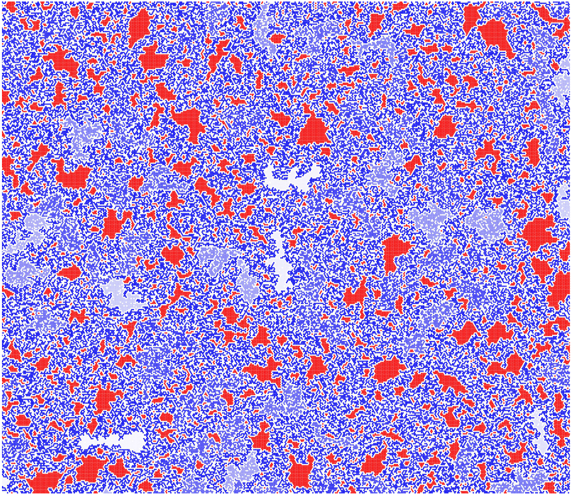

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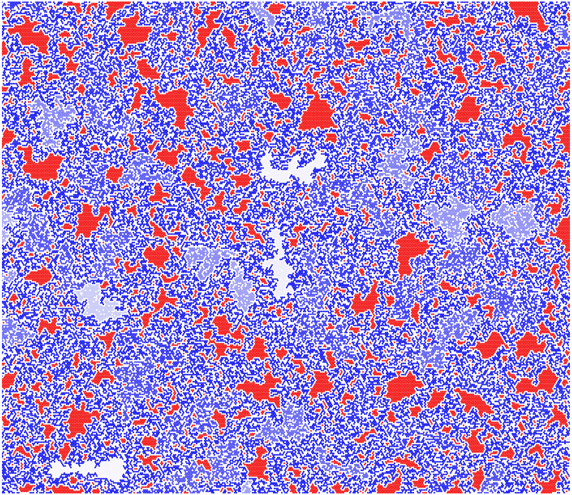

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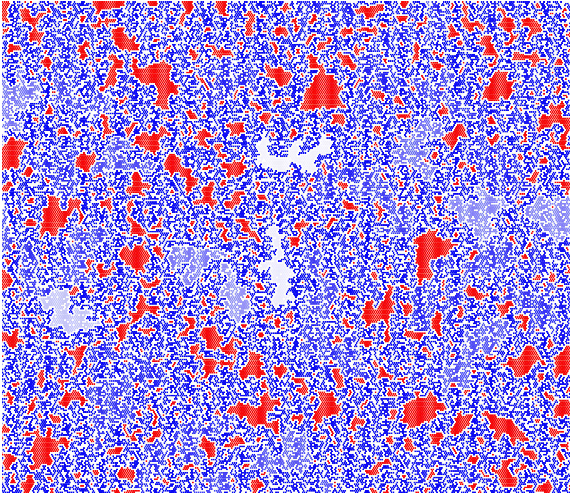

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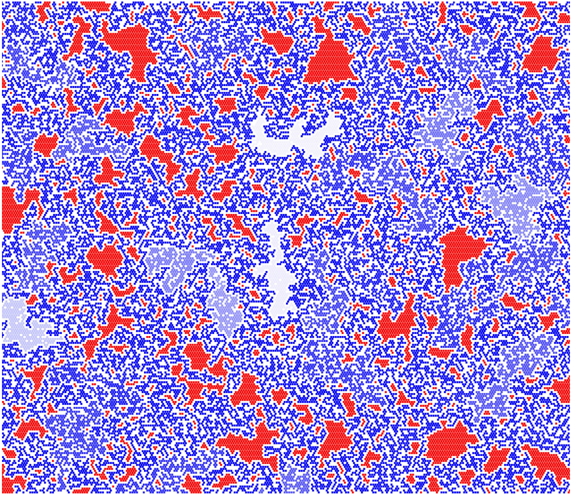

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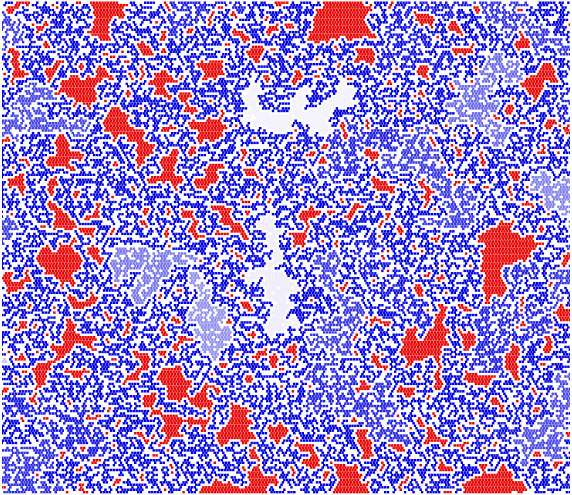

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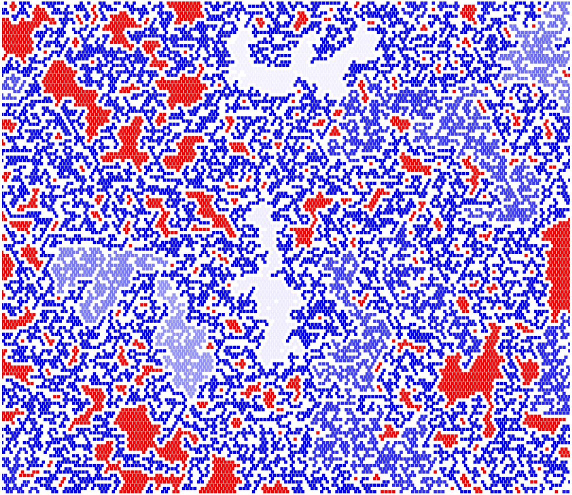

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  - **with recovery**: burnt vertices can become occupied again, at later birth times

Forest fire processes

- Relevant "macroscopic" behavior occurs around **critical time** t_c (defined by $1 - e^{-t_c} = p_c$): instance of self-organized criticality

[1] van den Berg, Kiss, N., *Two-dimensional volume-frozen percolation: deconcentration and prevalence of mesoscopic clusters*, Ann. Sci. ENS **51**, 1017–1084 (2018)

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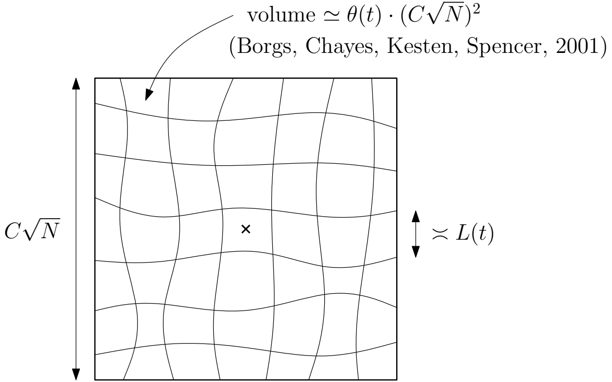
N-volume-frozen percolation

Box with side length $C\sqrt{N}$ ($C > 1$): for t just above t_c $\quad (1 - e^{-t_c} = p_c)$



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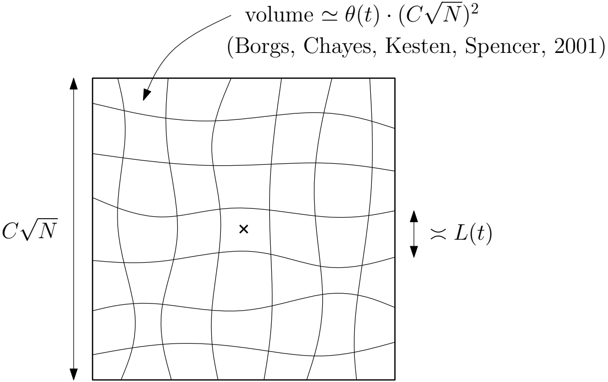

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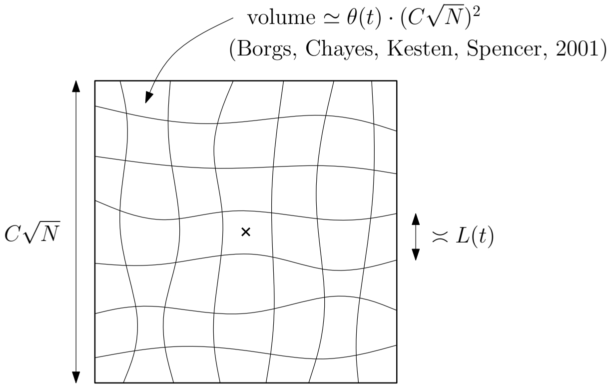

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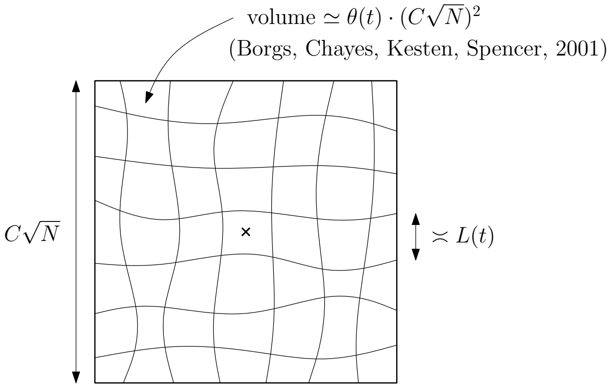

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- nothing else freezes: only 1 giant cluster freezes, "spanning" the box

Volume-frozen percolation

Proposition (van den Berg, N., 2014)

Run the process in a box with side length $C\sqrt{N}$ ($C > 1$):

$$\mathbb{P}_N^{B_{C\sqrt{N}}}(0 \text{ freezes}) \underset{N\to\infty}{\longrightarrow} \frac{1}{C^2}.$$

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No! $\sqrt{N}$ = first scale $m_1(N)$ in a sequence $(m_k(N))_{k\geq 1}$ of **exceptional scales**

$$m_k(N) = N^{\delta_k + o(1)},$$

with $\delta_1 = \frac{1}{2}$, and $\delta_{k+1} = \frac{1}{2} + \frac{5}{96}\delta_k \quad (\delta_k \nearrow \delta_\infty = \frac{48}{91})$.

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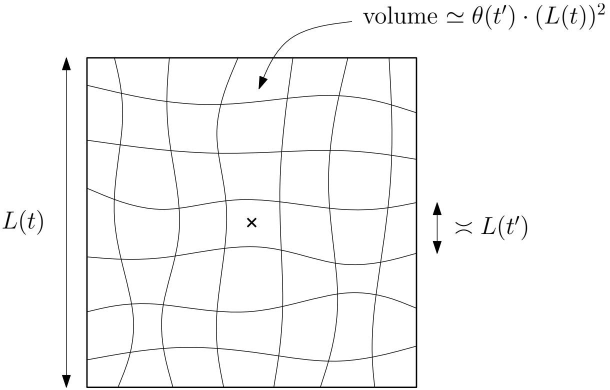

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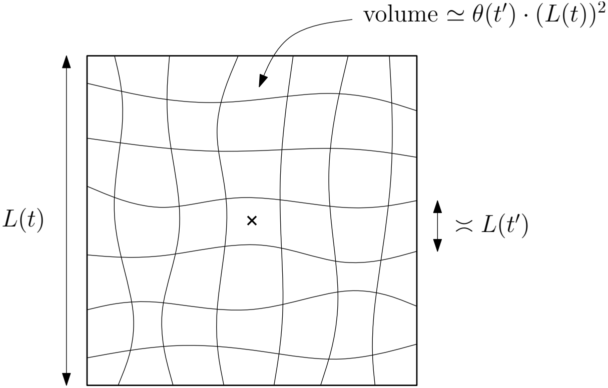

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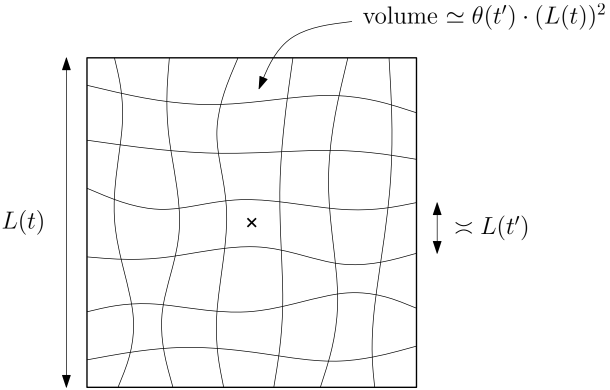

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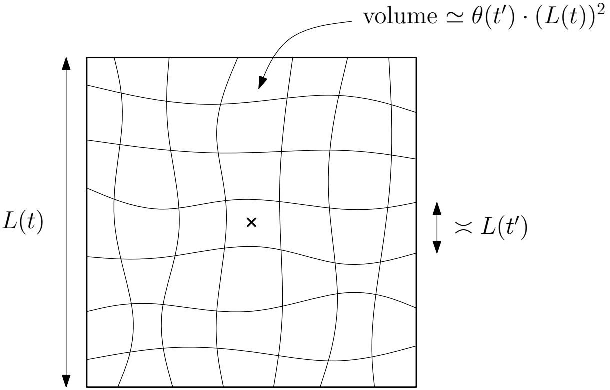
- leaves a hole around 0 with diameter $\asymp L(\hat{t})$,

Volume-frozen percolation

In a box with side length $m = L(t)$ ($t = t(N) \searrow t_c$): for t' just above t,



- freezes at a time very close to $\hat{t}$ s.t. $L(t)^2\theta(\hat{t}) = N$,
- leaves a hole around 0 with diameter $\asymp L(\hat{t})$,
- $\hat{m} = L(\hat{t})$ s.t. $m^2\pi_1(\hat{m}) \asymp N$.

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Exceptional scales: define $m_1(N) = \sqrt{N}$, then $m_2(N)$ s.t. $\hat{m}_2 = m_1$, then $m_3(N)$ s.t. $\hat{m}_3 = m_2$, and so on.

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$\rightarrow$ condition $m^2 \pi_1(m) \ll N$, i.e.

$$m \ll m_\infty(N) = N^{\delta_\infty + o(1)}$$

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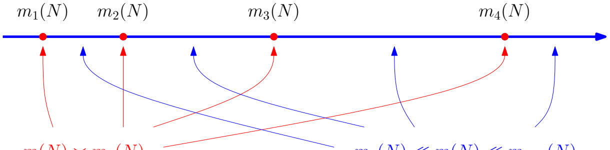
Theorem (van den Berg, N., 2014)

For N-volume-frozen percolation in box $B_{m(N)}$: as $N \to \infty$,



$m(N) \asymp m_k(N)$

$\liminf_{N \to \infty} \mathbb{P}_N^{B_{m(N)}}(0 \text{ freezes}) > 0$

freezing on (t_c, ∞)

$m_k(N) \ll m(N) \ll m_{k+1}(N)$

$\mathbb{P}_N^{B_{m(N)}}(0 \text{ freezes}) \underset{N \to \infty}{\longrightarrow} 0$

freezing only near t_c

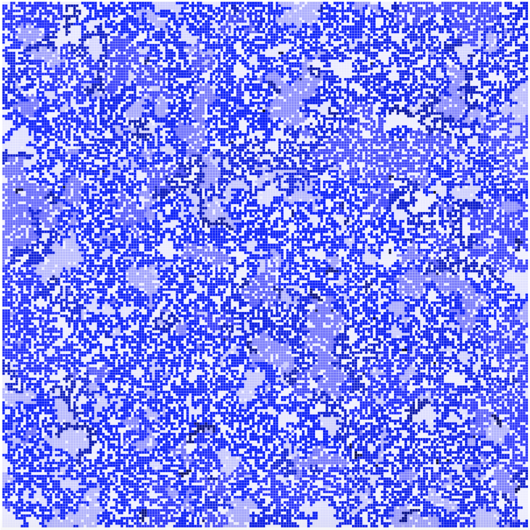
clusters in final configuration:

macroscopic (frozen / non-frozen)

microscopic (volume $O(1)$)

mesoscopic (volume $N^{\delta + o(1)}$)
$(0 < \delta < 1)$

Forest fires



Forest fire process without recovery, rate $\zeta = 0.01$

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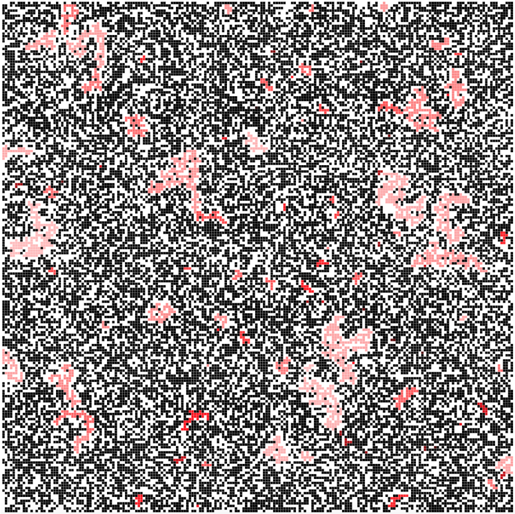
$\rightarrow$ we have to understand the effect of these **"impurities"** on the connectedness of the lattice

Forest fires



“impurities” created by fires before time $t_c - \varepsilon$ ($\varepsilon = 0.1$)

Forest fires

Connectedness of the forest at time $t_c - \varepsilon$?

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- if $m \lesssim m_k(\zeta)$, then $\zeta \ll m^{-\beta}$ for some $\beta = \beta_k > \frac{1}{\delta_\infty} = \alpha$

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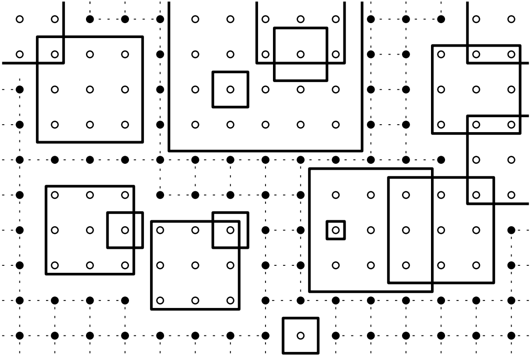
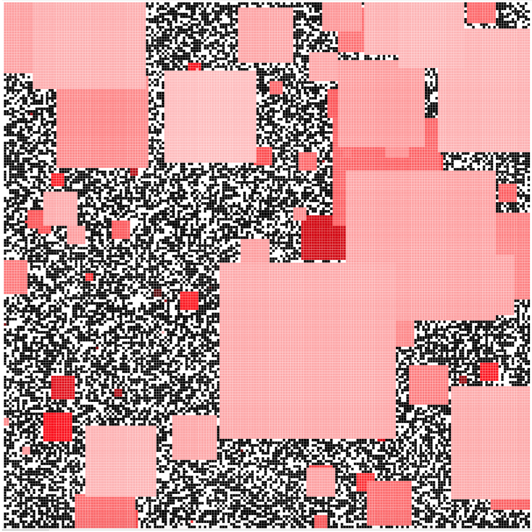


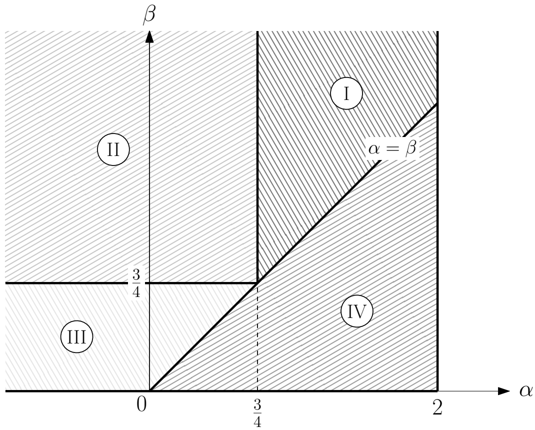
Heavy-tailed impurities

Percolation with heavy-tailed impurities: random environment



Heavy-tailed impurities

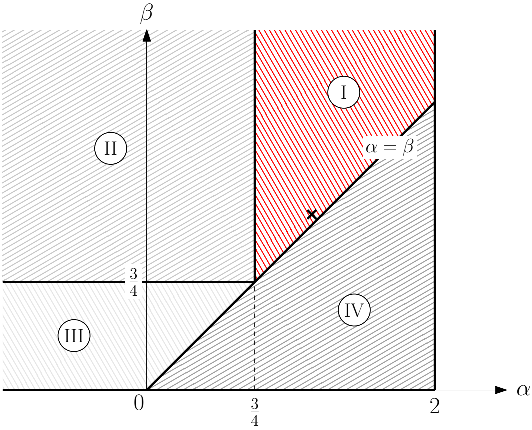
"Phase diagram" as α, β vary:



Heavy-tailed impurities

For forest fires, $\alpha = \frac{55}{48}$ and $\beta > \alpha$ (most interesting regime)

Note: impurities have density $m^{-(\beta-\alpha)}$, $\beta - \alpha$ arbitrarily small



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  $\rightarrow$ relies on inequality between arm exponents

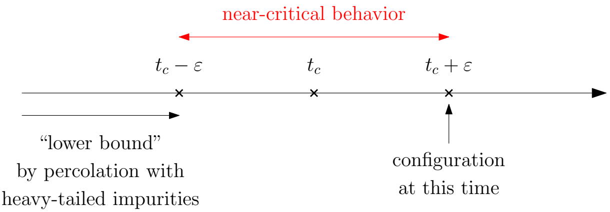
$$\alpha_4 \leq \alpha_2 + 1$$

Forest fires

Forest fire process at time $t_c + \varepsilon$, in a box with side length

$$M \gg m = L(t_c - \varepsilon) \asymp L(t_c + \varepsilon)$$

(typically, $m = \hat{M}$)



Forest fires

Conclusion:

- by studying **percolation with heavy tailed impurities**, we show that early fires do not perturb too much connectedness of the forest

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- at the moment, only very limited understanding of the long-term ($t \rightarrow \infty$) behavior

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End

Thank you!