The Tensor Track

Vincent Rivasseau
Laboratoire de Physique Théorique
CNRS UMR 8627, Université Paris-Sud,

2018 Nagoya International Workshop on the
Physics and Mathematics of Discrete Geometries,
Nagoya University, November 5 2018

The Tensor Track

There are many approaches to quantum gravity: string theory, M-theory, holography and AdS/CFT, higher spins, asymptotic safety, causal dynamical triangulations, loop quantum gravity, causal sets, Horava-Lifschitz gravity, ...

In 2010 Gurau discovered the tensor 1/N expansion, and in 2011, I coined the name "tensor track" for our program to explore this new and exciting framework, since

- it may lead to interesting results for quantum gravity because it is related to discretized random geometry pondered by Einstein-Hilbert action
- It should be useful in other domains: statistical mechanics, condensed matter, disordered systems, non-linear random flows, data analysis ...

The Tensor Track

There are many approaches to quantum gravity: string theory, M-theory, holography and AdS/CFT, higher spins, asymptotic safety, causal dynamical triangulations, loop quantum gravity, causal sets, Horava-Lifschitz gravity, ...

In 2010 Gurau discovered the tensor 1/N expansion, and in 2011, I coined the name "tensor track" for our program to explore this new and exciting framework, since

- it may lead to interesting results for quantum gravity because it is related to discretized random geometry pondered by Einstein-Hilbert action
- It should be useful in other domains: statistical mechanics, condensed matter, disordered systems, non-linear random flows, data analysis ...

The Tensor Track

There are many approaches to quantum gravity: string theory, M-theory, holography and AdS/CFT, higher spins, asymptotic safety, causal dynamical triangulations, loop quantum gravity, causal sets, Horava-Lifschitz gravity, ...

In 2010 Gurau discovered the tensor 1/N expansion, and in 2011, I coined the name "tensor track" for our program to explore this new and exciting framework, since

- it may lead to interesting results for quantum gravity because it is related to discretized random geometry pondered by Einstein-Hilbert action
- It should be useful in other domains: statistical mechanics, condensed matter, disordered systems, non-linear random flows, data analysis ...

The Tensor Track

There are many approaches to quantum gravity: string theory, M-theory, holography and AdS/CFT, higher spins, asymptotic safety, causal dynamical triangulations, loop quantum gravity, causal sets, Horava-Lifschitz gravity, ...

In 2010 Gurau discovered the tensor 1/N expansion, and in 2011, I coined the name "tensor track" for our program to explore this new and exciting framework, since

- it may lead to interesting results for quantum gravity because it is related to discretized random geometry pondered by Einstein-Hilbert action
- It should be useful in other domains: statistical mechanics, condensed matter, disordered systems, non-linear random flows, data analysis ...

The Tensor Track

There are many approaches to quantum gravity: string theory, M-theory, holography and AdS/CFT, higher spins, asymptotic safety, causal dynamical triangulations, loop quantum gravity, causal sets, Horava-Lifschitz gravity, ...

In 2010 Gurau discovered the tensor $1/N$ expansion, and in 2011, I coined the name "tensor track" for our program to explore this new and exciting framework, since

- it may lead to interesting results for quantum gravity because it is related to discretized random geometry pondered by Einstein-Hilbert action
- It should be useful in other domains: statistical mechanics, condensed matter, disordered systems, non-linear random flows, data analysis ...

Simpler situation in $d = 2$

Three approaches to two-dimensional quantum gravity

- Random Matrices and Surface Triangulations
- Liouville Theory
- Moduli Spaces of Riemann Surfaces

They are essentially equivalent (Miller-Sheffield...)

The tensor track is an attempt to extend the first approach to higher dimensions.

Simpler situation in $d = 2$

Three approaches to two-dimensional quantum gravity

- Random Matrices and Surface Triangulations
- Liouville Theory
- Moduli Spaces of Riemann Surfaces

They are essentially equivalent (Miller-Sheffield...)

The tensor track is an attempt to extend the first approach to higher dimensions.

Simpler situation in $d = 2$

Three approaches to two-dimensional quantum gravity

- Random Matrices and Surface Triangulations
- Liouville Theory
- Moduli Spaces of Riemann Surfaces

They are essentially equivalent (Miller-Sheffield...)

The tensor track is an attempt to extend the first approach to higher dimensions.

Simpler situation in $d = 2$

Three approaches to two-dimensional quantum gravity

- Random Matrices and Surface Triangulations
- Liouville Theory
- Moduli Spaces of Riemann Surfaces

They are essentially equivalent (Miller-Sheffield...)

The tensor track is an attempt to extend the first approach to higher dimensions.

Simpler situation in $d = 2$

Three approaches to two-dimensional quantum gravity

- Random Matrices and Surface Triangulations
- Liouville Theory
- Moduli Spaces of Riemann Surfaces

They are essentially equivalent (Miller-Sheffield...)

The tensor track is an attempt to extend the first approach to higher dimensions.

Simpler situation in $d = 2$

Three approaches to two-dimensional quantum gravity

- Random Matrices and Surface Triangulations
- Liouville Theory
- Moduli Spaces of Riemann Surfaces

They are essentially equivalent (Miller-Sheffield...)

The tensor track is an attempt to extend the first approach to higher dimensions.

Simpler situation in $d = 2$

Three approaches to two-dimensional quantum gravity

- Random Matrices and Surface Triangulations
- Liouville Theory
- Moduli Spaces of Riemann Surfaces

They are essentially equivalent (Miller-Sheffield...)

The tensor track is an attempt to extend the first approach to higher dimensions.

Discrete Random Geometries

Quantizing Gravity <=> Randomizing Geometry

$$Z \simeq \sum_S \int Dg \;\; e^{\int_S A_{EH}(g)}$$

- How to sum over metrics ($\int Dg$)?
- Should we sum over space time topologies ($\sum_S$)? Probably Yes (Maldacena...)
- Difficult problem, huge gauge invariance => discretize the problem

Discrete Random Geometries

Quantizing Gravity <=> Randomizing Geometry

$$Z \simeq \sum_S \int Dg \;\; e^{\int_S A_{EH}(g)}$$

- How to sum over metrics ($\int Dg$)?
- Should we sum over space time topologies ($\sum_S$)? Probably Yes (Maldacena...)
- Difficult problem, huge gauge invariance => discretize the problem

Discrete Random Geometries

Quantizing Gravity <=> Randomizing Geometry

$$Z \simeq \sum_S \int Dg \;\; e^{\int_S A_{EH}(g)}$$

- How to sum over metrics ($\int Dg$)?
- Should we sum over space time topologies ($\sum_S$)? Probably Yes (Maldacena...)
- Difficult problem, huge gauge invariance => discretize the problem

Discrete Random Geometries

Quantizing Gravity <=> Randomizing Geometry

$$Z \simeq \sum_S \int Dg \;\; e^{\int_S A_{EH}(g)}$$

- How to sum over metrics ($\int Dg$)?
- Should we sum over space time topologies ($\sum_S$)? Probably Yes (Maldacena...)
- Difficult problem, huge gauge invariance => discretize the problem

Discrete Random Geometries

Quantizing Gravity <=> Randomizing Geometry

$$Z \simeq \sum_S \int Dg \quad e^{\int_S A_{EH}(g)}$$

- How to sum over metrics ($\int Dg$)?
- Should we sum over space time topologies ($\sum_S$)? Probably Yes (Maldacena...)
- Difficult problem, huge gauge invariance => discretize the problem

Discrete Quantum Gravity, Matrix and Tensor Models

Regge calculus (60's) $->$ matrix models, 80's : David, Kazakov...
$->$ tensor models, 90's: Ambjorn, Durhuus, Jonsson, M. Gross, Sasakura...

Advantages

- No space-time to start with (background independence)
- No need for any gauge fixing (like in Wilson's lattice gauge theory)
- Can naturally include not only $\int Dg$ but also sum over topologies $(\sum_S)$

Discrete Quantum Gravity, Matrix and Tensor Models

Regge calculus (60's) $->$ matrix models, 80's : David, Kazakov...
$->$ tensor models, 90's: Ambjorn, Durhuus, Jonsson, M. Gross, Sasakura...

Advantages

- No space-time to start with (background independence)
- No need for any gauge fixing (like in Wilson's lattice gauge theory)
- Can naturally include not only $\int Dg$ but also sum over topologies ($\sum_S$)

Discrete Quantum Gravity, Matrix and Tensor Models

Regge calculus (60's) $->$ matrix models, 80's : David, Kazakov...
$->$ tensor models, 90's: Ambjorn, Durhuus, Jonsson, M. Gross, Sasakura...

Advantages

- No space-time to start with (background independence)
- No need for any gauge fixing (like in Wilson's lattice gauge theory)
- Can naturally include not only $\int Dg$ but also sum over topologies ($\sum_S$)

Discrete Quantum Gravity, Matrix and Tensor Models

Regge calculus (60's) $->$ matrix models, 80's : David, Kazakov...
$->$ tensor models, 90's: Ambjorn, Durhuus, Jonsson, M. Gross, Sasakura...

Advantages

- No space-time to start with (background independence)
- No need for any gauge fixing (like in Wilson's lattice gauge theory)
- Can naturally include not only $\int Dg$ but also sum over topologies ($\sum_S$)

Discrete Quantum Gravity, Matrix and Tensor Models

Regge calculus (60's) $->$ matrix models, 80's : David, Kazakov...
$->$ tensor models, 90's: Ambjorn, Durhuus, Jonsson, M. Gross, Sasakura...

Advantages

- No space-time to start with (background independence)
- No need for any gauge fixing (like in Wilson's lattice gauge theory)
- Can naturally include not only $\int Dg$ but also sum over topologies $(\sum_S)$

Discrete Quantum Gravity, Matrix and Tensor Models

Regge calculus (60's) $->$ matrix models, 80's : David, Kazakov...
$->$ tensor models, 90's: Ambjorn, Durhuus, Jonsson, M. Gross, Sasakura...

Advantages

- No space-time to start with (background independence)
- No need for any gauge fixing (like in Wilson's lattice gauge theory)
- Can naturally include not only $\int Dg$ but also sum over topologies $(\sum_S)$

$d = 2$: Matrix Models

$$Z_{\rm matrix} = \int dM \, e^{-\frac{1}{2}{\rm Tr} M^2 + \frac{\lambda}{\sqrt{N}} {\rm Tr} M^3}$$

- Feynman graphs are ribbon graphs, hence have faces. 't Hooft 1/N expansion is dominated by planar graphs

$$Z_{matrix} = \sum_{n,g} a_{n,g} \lambda^n N^{2-2g}, \quad 2g - 2 = V - E + F \tag{1.1}$$

- $\exists$ critical coupling λ_c. The $N \to \infty, \lambda \to \lambda_c$ single scaling limit leads to the brownian sphere.
- Double scaling limit, topological recursion... incorporate all genera.

$d = 2$: Matrix Models

$$Z_{\rm matrix} = \int dM \, e^{-\frac{1}{2}{\rm Tr} M^2 + \frac{\lambda}{\sqrt{N}} {\rm Tr} M^3}$$

- Feynman graphs are ribbon graphs, hence have faces. 't Hooft 1/N expansion is dominated by planar graphs

$$Z_{matrix} = \sum_{n,g} a_{n,g} \lambda^n N^{2-2g}, \quad 2g-2 = V - E + F \qquad (1.1)$$

- $\exists$ critical coupling λ_c. The $N \to \infty, \lambda \to \lambda_c$ single scaling limit leads to the brownian sphere.
- Double scaling limit, topological recursion... incorporate all genera.

$d = 2$: Matrix Models

$$Z_{\text{matrix}} = \int dM \, e^{-\frac{1}{2}\mathrm{Tr} M^2 + \frac{\lambda}{\sqrt{N}}\mathrm{Tr} M^3}$$

- Feynman graphs are ribbon graphs, hence have faces. 't Hooft 1/N expansion is dominated by planar graphs

$$Z_{matrix} = \sum_{n,g} a_{n,g} \lambda^n N^{2-2g}, \quad 2g - 2 = V - E + F \tag{1.1}$$

- $\exists$ critical coupling λ_c. The $N \to \infty, \lambda \to \lambda_c$ single scaling limit leads to the brownian sphere.
- Double scaling limit, topological recursion... incorporate all genera.

$d = 2$: Matrix Models

$$Z_{\rm matrix} = \int dM \, e^{-\frac{1}{2}{\rm Tr} M^2 + \frac{\lambda}{\sqrt{N}} {\rm Tr} M^3}$$

- Feynman graphs are ribbon graphs, hence have faces. 't Hooft 1/N expansion is dominated by planar graphs

$$Z_{matrix} = \sum_{n,g} a_{n,g} \lambda^n N^{2-2g}, \quad 2g - 2 = V - E + F \tag{1.1}$$

- $\exists$ critical coupling λ_c. The $N \to \infty, \lambda \to \lambda_c$ single scaling limit leads to the brownian sphere.
- Double scaling limit, topological recursion... incorporate all genera.

$d = 2$: Matrix Models

$$Z_{\text{matrix}} = \int dM \, e^{-\frac{1}{2}\mathrm{Tr}M^2 + \frac{\lambda}{\sqrt{N}}\mathrm{Tr}M^3}$$

- Feynman graphs are ribbon graphs, hence have faces. 't Hooft 1/N expansion is dominated by planar graphs

$$Z_{matrix} = \sum_{n,g} a_{n,g} \lambda^n N^{2-2g}, \quad 2g - 2 = V - E + F \tag{1.1}$$

- $\exists$ critical coupling λ_c. The $N \to \infty, \lambda \to \lambda_c$ single scaling limit leads to the brownian sphere.
- Double scaling limit, topological recursion... incorporate all genera.

Discretized Einstein-Hilbert Action in dimension d

On a triangulation with Q_d equilateral d-simplices and Q_{d-2} $(d-2)$-simplices:

$$A_{EH} = e^{\kappa_1 Q_{d-2} - \kappa_2 Q_d}.$$

On the dual graph G: $Q_d \to V$, number of vertices; $Q_{d-2} \to F$, number of faces, hence Regge action for equilateral simplices becomes

$$A_G(N) = \lambda^V N^F$$

the amplitudes of rank d tensor models.

The exact correspondence is (Ambjorn)

$$\ln N = \frac{vol(\sigma_{d-2})}{8G} = \frac{a_d}{G},$$

$$\ln \lambda = \frac{d}{16\pi G} vol(\sigma_{d-2}) \Big(\pi(d-1) - (d+1) \arccos \frac{1}{d} \Big) - 2\Lambda \, vol(\sigma_d)$$

Discretized Einstein-Hilbert Action in dimension d

On a triangulation with Q_d equilateral d-simplices and Q_{d-2} $(d-2)$-simplices:

$$A_{EH} = e^{\kappa_1 Q_{d-2} - \kappa_2 Q_d}.$$

On the dual graph G: $Q_d \to V$, number of vertices; $Q_{d-2} \to F$, number of faces, hence Regge action for equilateral simplices becomes

$$A_G(N) = \lambda^V N^F$$

the amplitudes of rank d tensor models.

The exact correspondence is (Ambjorn)

$$\ln N = \frac{vol(\sigma_{d-2})}{8G} = \frac{a_d}{G},$$

$$\ln \lambda = \frac{d}{16\pi G} vol(\sigma_{d-2})\Big(\pi(d-1) - (d+1)\arccos\frac{1}{d}\Big) - 2\Lambda\, vol(\sigma_d)$$

Discretized Einstein-Hilbert Action in dimension d

On a triangulation with Q_d equilateral d-simplices and Q_{d-2} $(d-2)$-simplices:

$$A_{EH} = e^{\kappa_1 Q_{d-2} - \kappa_2 Q_d}.$$

On the dual graph G: $Q_d \to V$, number of vertices; $Q_{d-2} \to F$, number of faces, hence Regge action for equilateral simplices becomes

$$A_G(N) = \lambda^V N^F$$

the amplitudes of rank d tensor models.

The exact correspondence is (Ambjorn)

$$\ln N = \frac{vol(\sigma_{d-2})}{8G} = \frac{a_d}{G},$$

$$\ln \lambda = \frac{d}{16\pi G} vol(\sigma_{d-2})\Big(\pi(d-1) - (d+1)\arccos\frac{1}{d}\Big) - 2\Lambda\, vol(\sigma_d)$$

Discretized Einstein-Hilbert Action in dimension d

On a triangulation with Q_d equilateral d-simplices and Q_{d-2} $(d-2)$-simplices:

$$A_{EH} = e^{\kappa_1 Q_{d-2} - \kappa_2 Q_d}.$$

On the dual graph G: $Q_d \to V$, number of vertices; $Q_{d-2} \to F$, number of faces, hence Regge action for equilateral simplices becomes

$$A_G(N) = \lambda^V N^F$$

the amplitudes of rank d tensor models.

The exact correspondence is (Ambjorn)

$$\ln N = \frac{vol(\sigma_{d-2})}{8G} = \frac{a_d}{G},$$

$$\ln \lambda = \frac{d}{16\pi G} vol(\sigma_{d-2})\Big(\pi(d-1) - (d+1)\arccos\frac{1}{d}\Big) - 2\Lambda\, vol(\sigma_d)$$

Discretized Einstein-Hilbert Action in dimension d

On a triangulation with Q_d equilateral d-simplices and Q_{d-2} $(d-2)$-simplices:

$$A_{EH} = e^{\kappa_1 Q_{d-2} - \kappa_2 Q_d}.$$

On the dual graph G: $Q_d \to V$, number of vertices; $Q_{d-2} \to F$, number of faces, hence Regge action for equilateral simplices becomes

$$A_G(N) = \lambda^V N^F$$

the amplitudes of rank d tensor models.

The exact correspondence is (Ambjorn)

$$\begin{aligned} \ln N &= \frac{vol(\sigma_{d-2})}{8G} = \frac{a_d}{G} , \\ \ln \lambda &= \frac{d}{16\pi G} vol(\sigma_{d-2}) \Big(\pi(d-1) - (d+1) \arccos \frac{1}{d} \Big) - 2\Lambda\, vol(\sigma_d) \end{aligned}$$

Still Problems ($d \geq 3$)

However the early 90's tensor models met some problems:

- no simple homology theory,
- no analog of 't Hooft 1/N expansion,
- very singular spaces seem to dominate the sum.

Still Problems ($d \geq 3$)

However the early 90's tensor models met some problems:

- no simple homology theory,
- no analog of 't Hooft 1/N expansion,
- very singular spaces seem to dominate the sum.

Still Problems ($d \geq 3$)

However the early 90's tensor models met some problems:

- no simple homology theory,
- no analog of 't Hooft 1/N expansion,
- very singular spaces seem to dominate the sum.

Still Problems ($d \geq 3$)

However the early 90's tensor models met some problems:

- no simple homology theory,
- no analog of 't Hooft 1/N expansion,
- very singular spaces seem to dominate the sum.

Tensor $1/N$ expansion

The introduction of unsymmetrized tensors by Gurau (2009) and the subsequent discovery of the tensor $1/N$ expansion (2010) solved these problems

- canonical notion of faces and full d-homology,
- $1/N$ expansion, which is not topological,
- less singular spaces (only local conical singularities).

Tensor $1/N$ expansion

The introduction of unsymmetrized tensors by Gurau (2009) and the subsequent discovery of the tensor $1/N$ expansion (2010) solved these problems

- canonical notion of faces and full d-homology,
- $1/N$ expansion, which is not topological,
- less singular spaces (only local conical singularities).

Tensor $1/N$ expansion

The introduction of unsymmetrized tensors by Gurau (2009) and the subsequent discovery of the tensor $1/N$ expansion (2010) solved these problems

- canonical notion of faces and full d-homology,
- $1/N$ expansion, which is not topological,
- less singular spaces (only local conical singularities).

Tensor $1/N$ expansion

The introduction of unsymmetrized tensors by Gurau (2009) and the subsequent discovery of the tensor $1/N$ expansion (2010) solved these problems

- canonical notion of faces and full d-homology,
- $1/N$ expansion, which is not topological,
- less singular spaces (only local conical singularities).

Random Vectors, Matrices, Tensors

Random Vectors $\subset$ Random Matrices $\subset$ Random Tensors

- Each class is richer than the previous one, having more and more invariants
- Each class has some universal aspects and a different $1/N$ expansion
- Each class is connected to the discretized random geometric approach to quantum gravity, where roughly speaking rank $\simeq$ dimension
- Each class has or should have many concrete applications

Random Vectors, Matrices, Tensors

Random Vectors $\subset$ Random Matrices $\subset$ Random Tensors

- Each class is richer than the previous one, having more and more invariants
- Each class has some universal aspects and a different $1/N$ expansion
- Each class is connected to the discretized random geometric approach to quantum gravity, where roughly speaking rank $\simeq$ dimension
- Each class has or should have many concrete applications

Random Vectors, Matrices, Tensors

Random Vectors $\subset$ Random Matrices $\subset$ Random Tensors

- Each class is richer than the previous one, having more and more invariants
- Each class has some universal aspects and a different $1/N$ expansion
- Each class is connected to the discretized random geometric approach to quantum gravity, where roughly speaking rank $\simeq$ dimension
- Each class has or should have many concrete applications

Random Vectors, Matrices, Tensors

Random Vectors $\subset$ Random Matrices $\subset$ Random Tensors

- Each class is richer than the previous one, having more and more invariants
- Each class has some universal aspects and a different $1/N$ expansion
- Each class is connected to the discretized random geometric approach to quantum gravity, where roughly speaking rank $\simeq$ dimension
- Each class has or should have many concrete applications

Random Vectors, Matrices, Tensors

Random Vectors $\subset$ Random Matrices $\subset$ Random Tensors

- Each class is richer than the previous one, having more and more invariants
- Each class has some universal aspects and a different $1/N$ expansion
- Each class is connected to the discretized random geometric approach to quantum gravity, where roughly speaking rank $\simeq$ dimension
- Each class has or should have many concrete applications

Random Vectors, Matrices, Tensors

Random Vectors $\subset$ Random Matrices $\subset$ Random Tensors

- Each class is richer than the previous one, having more and more invariants
- Each class has some universal aspects and a different $1/N$ expansion
- Each class is connected to the discretized random geometric approach to quantum gravity, where roughly speaking rank $\simeq$ dimension
- Each class has or should have many concrete applications

Random Vectors, Matrices, Tensors

Random Vectors $\subset$ Random Matrices $\subset$ Random Tensors

- Each class is richer than the previous one, having more and more invariants
- Each class has some universal aspects and a different $1/N$ expansion
- Each class is connected to the discretized random geometric approach to quantum gravity, where roughly speaking rank $\simeq$ dimension
- Each class has or should have many concrete applications

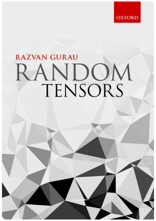
Random Vectors, Matrices, Tensors

Random Vectors $\subset$ Random Matrices $\subset$ Random Tensors

- Each class is richer than the previous one, having more and more invariants
- Each class has some universal aspects and a different $1/N$ expansion
- Each class is connected to the discretized random geometric approach to quantum gravity, where roughly speaking rank $\simeq$ dimension
- Each class has or should have many concrete applications

Reviews on Random Tensors

To learn the subject:

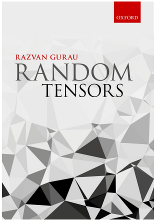


More specialized aspects:

- Sigma 2016 Special Issue on Tensor Models, Formalism and Applications
- Klebanov, Popov and Tarnopolsky, TASI Lectures on Large N Tensor Models

Reviews on Random Tensors

To learn the subject:

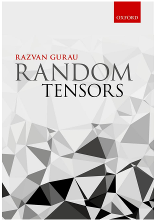


More specialized aspects:

- Sigma 2016 Special Issue on Tensor Models, Formalism and Applications
- Klebanov, Popov and Tarnopolsky, TASI Lectures on Large N Tensor Models

Reviews on Random Tensors

To learn the subject:

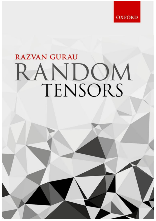


More specialized aspects:

- Sigma 2016 Special Issue on Tensor Models, Formalism and Applications
- Klebanov, Popov and Tarnopolsky, TASI Lectures on Large N Tensor Models

Reviews on Random Tensors

To learn the subject:



More specialized aspects:

- Sigma 2016 Special Issue on Tensor Models, Formalism and Applications
- Klebanov, Popov and Tarnopolsky, TASI Lectures on Large N Tensor Models

The Colored $U(N)$ Tensor Model

- uses $D+1$ random tensors;
- its Feynman graphs are dual to simplicial (orientable) triangulations

Probability measure

$$d\nu = \frac{1}{Z} \prod_{i,n_i} \frac{dT^i_{n_i} d\bar{T}^i_{\bar{n}_i}}{2\pi} e^{-S(T,\bar{T})}$$

$$S = \sum_{i=0}^{D} \bar{T}^i \cdot T^i + \frac{\lambda}{N^{D(D-1)/4}} \sum_{\{n\}} \prod_{i=0}^{D} T^i_{n_i} \prod_{i<j} \delta_{n^{ij}, n^{ji}} + cc$$

where $\sum_{\vec{n}}$ denotes the sum over all indices n_{ij} from 1 to N. The $\frac{(D+1)D}{2}$ identifying δ functions follow the pattern of edges of the K_{D+1} complete graph on $D+1$ vertices.

The Colored $U(N)$ Tensor Model

- uses $D+1$ random tensors;
- its Feynman graphs are dual to simplicial (orientable) triangulations

Probability measure

$$d\nu = \frac{1}{Z} \prod_{i,n_i} \frac{dT^i_{n_i} d\bar{T}^i_{\bar{n}_i}}{2\pi} e^{-S(T,\bar{T})}$$

$$S = \sum_{i=0}^{D} \bar{T}^i \cdot T^i + \frac{\lambda}{N^{D(D-1)/4}} \sum_{\{n\}} \prod_{i=0}^{D} T^i_{n_i} \prod_{i<j} \delta_{n^{ij}, n^{ji}} + cc$$

where $\sum_{\vec{n}}$ denotes the sum over all indices n_{ij} from 1 to N. The $\frac{(D+1)D}{2}$ identifying δ functions follow the pattern of edges of the K_{D+1} complete graph on $D+1$ vertices.

The Colored $U(N)$ Tensor Model

- uses $D+1$ random tensors;
- its Feynman graphs are dual to simplicial (orientable) triangulations

Probability measure

$$d\nu = \frac{1}{Z}\prod_{i,n_i}\frac{dT^i_{n_i}\,d\bar{T}^i_{\bar{n}_i}}{2\pi}e^{-S(T,\bar{T})}$$

$$S = \sum_{i=0}^{D}\bar{T}^i\cdot T^i + \frac{\lambda}{N^{D(D-1)/4}}\sum_{\{n\}}\prod_{i=0}^{D}T^i_{n_i}\prod_{i<j}\delta_{n^{ij},n^{ji}} + cc$$

where $\sum_{\vec{n}}$ denotes the sum over all indices n_{ij} from 1 to N. The $\frac{(D+1)D}{2}$ identifying δ functions follow the pattern of edges of the K_{D+1} complete graph on $D+1$ vertices.

The Colored $U(N)$ Tensor Model

- uses $D+1$ random tensors;
- its Feynman graphs are dual to simplicial (orientable) triangulations

Probability measure

$$d\nu = \frac{1}{Z} \prod_{i,n_i} \frac{dT^i_{n_i} d\bar{T}^i_{\bar{n}_i}}{2\pi} e^{-S(T,\bar{T})}$$

$$S = \sum_{i=0}^{D} \bar{T}^i \cdot T^i + \frac{\lambda}{N^{D(D-1)/4}} \sum_{\{n\}} \prod_{i=0}^{D} T^i_{n_i} \prod_{i<j} \delta_{n^{ij},n^{ji}} + cc$$

where $\sum_{\vec{n}}$ denotes the sum over all indices n_{ij} from 1 to N. The $\frac{(D+1)D}{2}$ identifying δ functions follow the pattern of edges of the K_{D+1} complete graph on $D+1$ vertices.

The Colored $U(N)$ Tensor Model

- uses $D+1$ random tensors;
- its Feynman graphs are dual to simplicial (orientable) triangulations

Probability measure

$$d\nu = \frac{1}{Z} \prod_{i,n_i} \frac{dT^i_{n_i} d\bar{T}^i_{\bar{n}_i}}{2\pi} e^{-S(T,\bar{T})}$$

$$S = \sum_{i=0}^{D} \bar{T}^i \cdot T^i + \frac{\lambda}{N^{D(D-1)/4}} \sum_{\{n\}} \prod_{i=0}^{D} T^i_{n_i} \prod_{i<j} \delta_{n^{ij}, n^{ji}} + cc$$

where $\sum_{\vec{n}}$ denotes the sum over all indices n_{ij} from 1 to N. The $\frac{(D+1)D}{2}$ identifying δ functions follow the pattern of edges of the K_{D+1} complete graph on $D+1$ vertices.

The Colored $U(N)$ Tensor Model

- uses $D+1$ random tensors;
- its Feynman graphs are dual to simplicial (orientable) triangulations

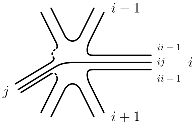
Probability measure

$$d\nu = \frac{1}{Z} \prod_{i,n_i} \frac{dT^i_{n_i} d\bar{T}^i_{\bar{n}_i}}{2\pi} e^{-S(T,\bar{T})}$$

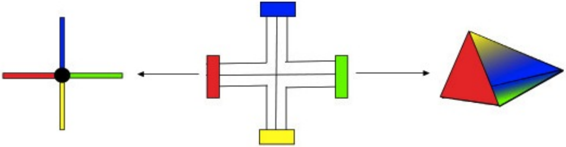
$$S = \sum_{i=0}^{D} \bar{T}^i \cdot T^i + \frac{\lambda}{N^{D(D-1)/4}} \sum_{\{n\}} \prod_{i=0}^{D} T^i_{n_i} \prod_{i<j} \delta_{n^{ij},n^{ji}} + cc$$

where $\sum_{\vec{n}}$ denotes the sum over all indices n_{ij} from 1 to N. The $\frac{(D+1)D}{2}$ identifying δ functions follow the pattern of edges of the K_{D+1} complete graph on $D+1$ vertices.

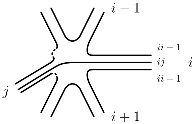
Feynman Graphs



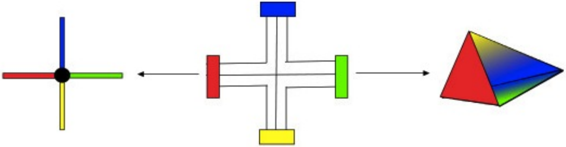
- Colors can conveniently encode strands
- and gluing rules for dual triangulations



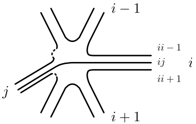
Feynman Graphs



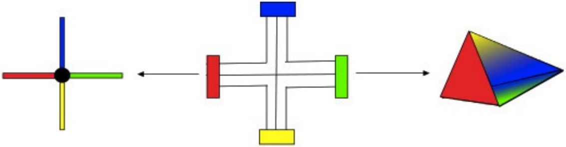
- Colors can conveniently encode strands
- and gluing rules for dual triangulations



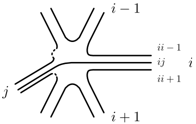
Feynman Graphs



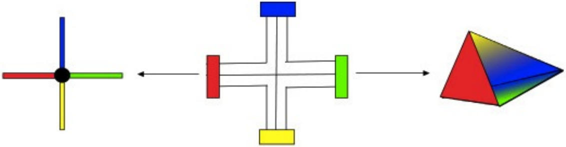
- Colors can conveniently encode strands
- and gluing rules for dual triangulations



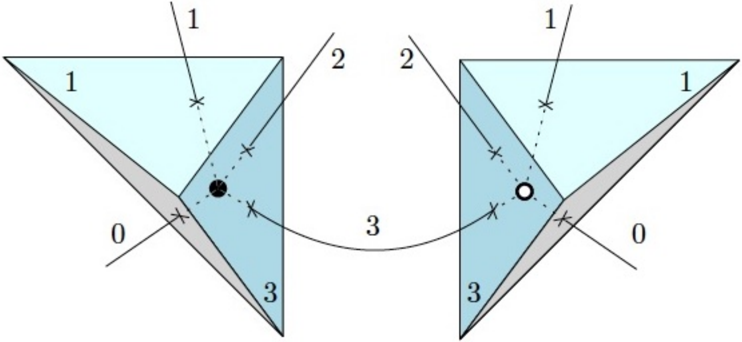
Feynman Graphs



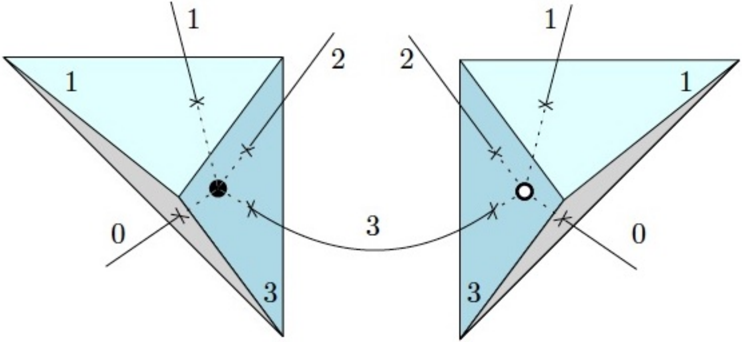
- Colors can conveniently encode strands
- and gluing rules for dual triangulations



Feynman Graphs



Feynman Graphs



D-Homology

- For D-regular edge-colored graphs there is a simple canonical definition of faces
- k-dimensional objects = connected components with k colors
- hence edges = 1-colored components, faces = 2-colored components
- faces exist without any embedding in a surface!

D-Homology

- For D-regular edge-colored graphs there is a simple canonical definition of faces
- k-dimensional objects = connected components with k colors
- hence edges = 1-colored components, faces = 2-colored components
- faces exist without any embedding in a surface!

D-Homology

- For D-regular edge-colored graphs there is a simple canonical definition of faces
- k-dimensional objects = connected components with k colors
- hence edges = 1-colored components, faces = 2-colored components
- faces exist without any embedding in a surface!

D-Homology

- For D-regular edge-colored graphs there is a simple canonical definition of faces
- k-dimensional objects = connected components with k colors
- hence edges = 1-colored components, faces = 2-colored components
- faces exist without any embedding in a surface!

D-Homology

- For D-regular edge-colored graphs there is a simple canonical definition of faces
- k-dimensional objects = connected components with k colors
- hence edges = 1-colored components, faces = 2-colored components
- faces exist without any embedding in a surface!

(Un)-colored Tensor Models

Basic objects: $U(N)^{\otimes d}$ tensor invariants = regular d-edge-colored connected bipartite graphs

- are dual to colored triangulations
- are the interactions (vertices) of rank-d random tensors
- are the observables of rank-d random tensors
- are the Feynman graphs of rank-$d-1$ random tensors

(Un)-colored Tensor Models

Basic objects: $U(N)^{\otimes d}$ tensor invariants = regular d-edge-colored connected bipartite graphs

- are dual to colored triangulations
- are the interactions (vertices) of rank-d random tensors
- are the observables of rank-d random tensors
- are the Feynman graphs of rank-$d-1$ random tensors

(Un)-colored Tensor Models

Basic objects: $U(N)^{\otimes d}$ tensor invariants = regular d-edge-colored connected bipartite graphs

- are dual to colored triangulations
- are the interactions (vertices) of rank-d random tensors
- are the observables of rank-d random tensors
- are the Feynman graphs of rank-$d-1$ random tensors

(Un)-colored Tensor Models

Basic objects: $U(N)^{\otimes d}$ tensor invariants = regular d-edge-colored connected bipartite graphs

- are dual to colored triangulations
- are the interactions (vertices) of rank-d random tensors
- are the observables of rank-d random tensors
- are the Feynman graphs of rank-$d-1$ random tensors

(Un)-colored Tensor Models

Basic objects: $U(N)^{\otimes d}$ tensor invariants = regular d-edge-colored connected bipartite graphs

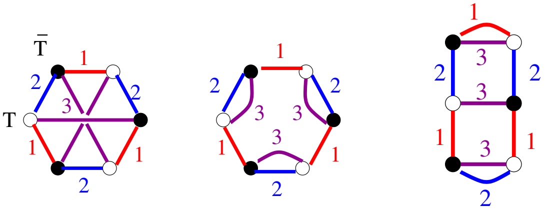
- are dual to colored triangulations
- are the interactions (vertices) of rank-d random tensors
- are the observables of rank-d random tensors
- are the Feynman graphs of rank-$d-1$ random tensors

(Un)-colored Tensor Models

Basic objects: $U(N)^{\otimes d}$ tensor invariants = regular d-edge-colored connected bipartite graphs

- are dual to colored triangulations
- are the interactions (vertices) of rank-d random tensors
- are the observables of rank-d random tensors
- are the Feynman graphs of rank-$d-1$ random tensors

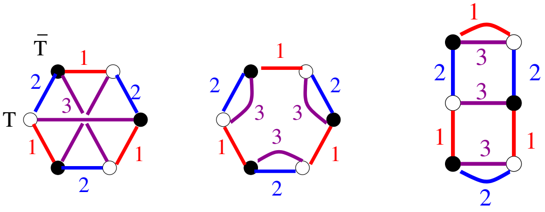
Tensor Invariants



Tensor invariants can be counted as equivalence classes of permutations (J. Ben Geloun and S. Ramgoolam)

$$
\begin{aligned}
Z_1^c(n) &= 1,0,0,0,0,\ldots && \bar{\Phi}\cdot\Phi \\
Z_2^c(n) &= 1,1,1,1,1,1,1\ldots && \mathrm{Tr}(MM^\dagger)^n \\
Z_3^c(n) &= 1,3,7,26,97,624,4163\ldots \\
Z_4^c(n) &= 1,7,41,604,13753\ldots
\end{aligned}
$$

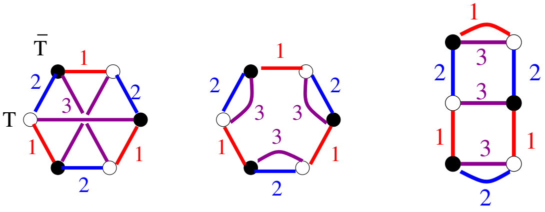
Tensor Invariants



Tensor invariants can be counted as equivalence classes of permutations (J. Ben Geloun and S. Ramgoolam)

$$
\begin{aligned}
Z_1^c(n) &= 1,0,0,0,0,\ldots & \bar{\Phi}\cdot\Phi \\
Z_2^c(n) &= 1,1,1,1,1,1,1\ldots & \mathrm{Tr}(MM^\dagger)^n \\
Z_3^c(n) &= 1,3,7,26,97,624,4163\ldots \\
Z_4^c(n) &= 1,7,41,604,13753\ldots
\end{aligned}
$$

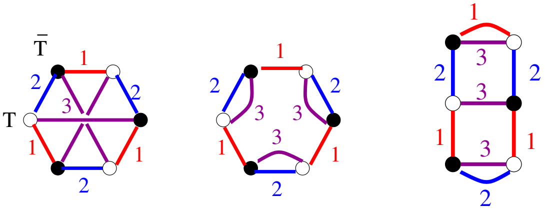
Tensor Invariants



Tensor invariants can be counted as equivalence classes of permutations (J. Ben Geloun and S. Ramgoolam)

$$
\begin{aligned}
Z_1^c(n) &= 1, 0, 0, 0, 0, \ldots && \bar{\Phi} \cdot \Phi \\
Z_2^c(n) &= 1, 1, 1, 1, 1, 1, 1\ldots && \mathrm{Tr}(MM^\dagger)^n \\
Z_3^c(n) &= 1, 3, 7, 26, 97, 624, 4163\ldots \\
Z_4^c(n) &= 1, 7, 41, 604, 13753\ldots
\end{aligned}
$$

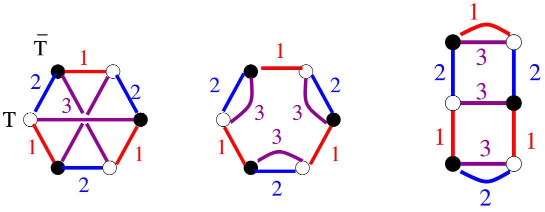
Tensor Invariants



Tensor invariants can be counted as equivalence classes of permutations (J. Ben Geloun and S. Ramgoolam)

$$
\begin{aligned}
Z_1^c(n) &= 1, 0, 0, 0, 0, \ldots && \bar{\Phi} \cdot \Phi \\
Z_2^c(n) &= 1, 1, 1, 1, 1, 1, 1\ldots && \mathrm{Tr}(MM^\dagger)^n \\
Z_3^c(n) &= 1, 3, 7, 26, 97, 624, 4163\ldots \\
Z_4^c(n) &= 1, 7, 41, 604, 13753\ldots
\end{aligned}
$$

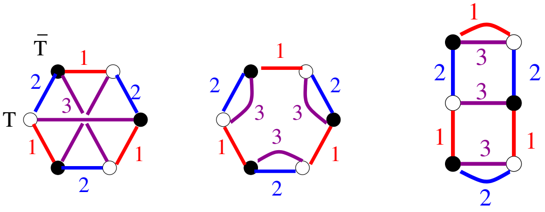
Tensor Invariants



Tensor invariants can be counted as equivalence classes of permutations (J. Ben Geloun and S. Ramgoolam)

$$
\begin{aligned}
Z_1^c(n) &= 1, 0, 0, 0, 0, ... & \bar{\Phi} \cdot \Phi \\
Z_2^c(n) &= 1, 1, 1, 1, 1, 1, 1... & \mathrm{Tr}(MM^\dagger)^n \\
Z_3^c(n) &= 1, 3, 7, 26, 97, 624, 4163... \\
Z_4^c(n) &= 1, 7, 41, 604, 13753...
\end{aligned}
$$

Tensor Invariants



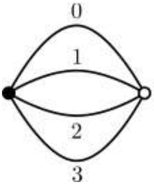
Tensor invariants can be counted as equivalence classes of permutations (J. Ben Geloun and S. Ramgoolam)

$$
\begin{aligned}
Z_1^c(n) &= 1, 0, 0, 0, 0, \ldots && \bar{\Phi} \cdot \Phi \\
Z_2^c(n) &= 1, 1, 1, 1, 1, 1, 1\ldots && \mathrm{Tr}(MM^\dagger)^n \\
Z_3^c(n) &= 1, 3, 7, 26, 97, 624, 4163\ldots \\
Z_4^c(n) &= 1, 7, 41, 604, 13753\ldots
\end{aligned}
$$

Melonic Graphs

Elementary vacuum melon: two vertices, $D+1$ edges:

$=$



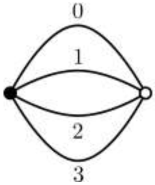
2-point elementary melon of color $i \in \{0, 1, \cdots, D\}$: cut the line of color i.

Definition (Vacuum) melonic graphs are the graphs obtained from the elementary vacuum melon by finitely many recursive insertions of a 2-point elementary melon of color $i \in \{0, 1, \cdots, D\}$ on any edge of the *same* color i.

Melonic Graphs

Elementary vacuum melon: two vertices, $D+1$ edges:

=



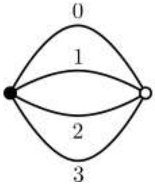
2-point elementary melon of color $i \in \{0, 1, \cdots, D\}$: cut the line of color i.

Definition (Vacuum) melonic graphs are the graphs obtained from the elementary vacuum melon by finitely many recursive insertions of a 2-point elementary melon of color $i \in \{0, 1, \cdots, D\}$ on any edge of the *same* color i.

Melonic Graphs

Elementary vacuum melon: two vertices, $D + 1$ edges:

$=$



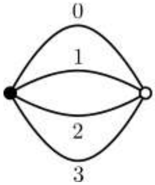
2-point elementary melon of color $i \in \{0, 1, \cdots, D\}$: cut the line of color i.

Definition (Vacuum) melonic graphs are the graphs obtained from the elementary vacuum melon by finitely many recursive insertions of a 2-point elementary melon of color $i \in \{0, 1, \cdots, D\}$ on any edge of the *same* color i.

Melonic Graphs

Elementary vacuum melon: two vertices, $D+1$ edges:

$=$



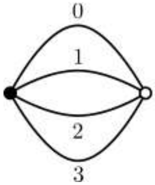
2-point elementary melon of color $i \in \{0, 1, \cdots, D\}$: cut the line of color i.

Definition (Vacuum) melonic graphs are the graphs obtained from the elementary vacuum melon by finitely many recursive insertions of a 2-point elementary melon of color $i \in \{0, 1, \cdots, D\}$ on any edge of the *same* color i.

Melonic Graphs

Elementary vacuum melon: two vertices, $D+1$ edges:

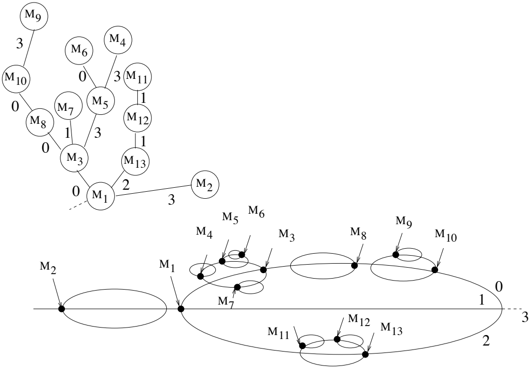
$=$



2-point elementary melon of color $i \in \{0, 1, \cdots, D\}$: cut the line of color i.

Definition (Vacuum) melonic graphs are the graphs obtained from the elementary vacuum melon by finitely many recursive insertions of a 2-point elementary melon of color $i \in \{0, 1, \cdots, D\}$ on any edge of the *same* color i.

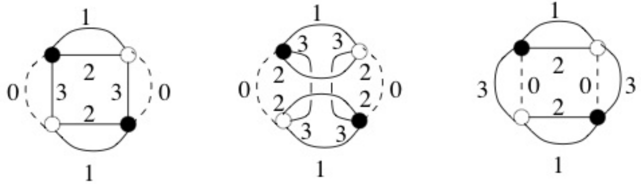
The Result of the Recursion



Jackets, Degree, $1/N$ Expansion

Jacket $J =$ color cycle up to orientation ($D!/2$ at rank D)

Defines a ribbon graph G_J with same number of lines and vertices than G. This ribbon graph has a genus g_J.



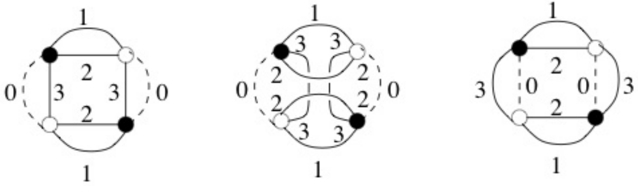
$A(G) \propto N^{D - \frac{2}{D!}\omega(G)}$, where $\omega = \sum_J g(J) \geq 0$, the Gurau degree, governs the expansion.

For $D \geq 3$ this degree is not a topological invariant of the space dual to G.

Jackets, Degree, $1/N$ Expansion

Jacket J = color cycle up to orientation ($D!/2$ at rank D)

Defines a ribbon graph G_J with same number of lines and vertices than G. This ribbon graph has a genus g_J.



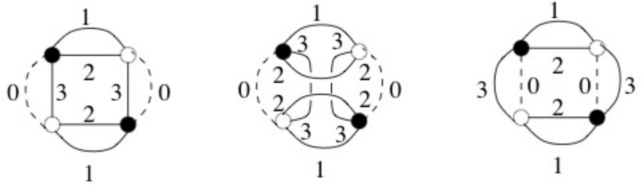
$A(G) \propto N^{D-\frac{2}{D!}\omega(G)}$, where $\omega = \sum_J g(J) \geq 0$, the Gurau degree, governs the expansion.

For $D \geq 3$ this degree is not a topological invariant of the space dual to G.

Jackets, Degree, $1/N$ Expansion

Jacket J = color cycle up to orientation ($D!/2$ at rank D)

Defines a ribbon graph G_J with same number of lines and vertices than G. This ribbon graph has a genus g_J.



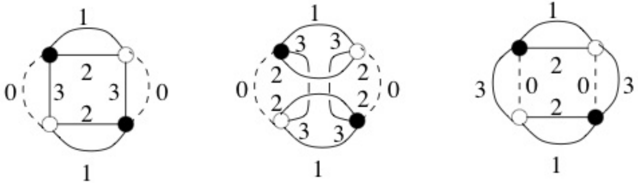
$A(G) \propto N^{D-\frac{2}{D!}\omega(G)}$, where $\omega = \sum_J g(J) \geq 0$, the Gurau degree, governs the expansion.

For $D \geq 3$ this degree is not a topological invariant of the space dual to G.

Jackets, Degree, $1/N$ Expansion

Jacket J = color cycle up to orientation ($D!/2$ at rank D)

Defines a ribbon graph G_J with same number of lines and vertices than G. This ribbon graph has a genus g_J.



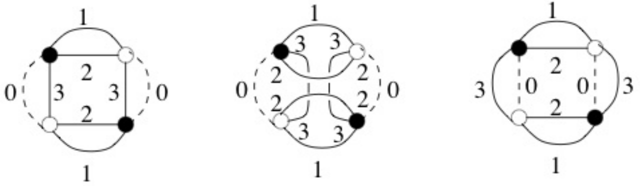
$A(G) \propto N^{D-\frac{2}{D!}\omega(G)}$, where $\omega = \sum_J g(J) \geq 0$, the Gurau degree, governs the expansion.

For $D \geq 3$ this degree is not a topological invariant of the space dual to G.

Jackets, Degree, $1/N$ Expansion

Jacket $J =$ color cycle up to orientation ($D!/2$ at rank D)

Defines a ribbon graph G_J with same number of lines and vertices than G. This ribbon graph has a genus g_J.



$A(G) \propto N^{D - \frac{2}{D!}\omega(G)}$, where $\omega = \sum_J g(J) \geq 0$, the Gurau degree, governs the expansion.

For $D \geq 3$ this degree is not a topological invariant of the space dual to G.

Counting Faces with Jackets

Each face f_{ij} belongs to $(D-1)!$ jackets (the ones in which i and j are adjacent).

$$2 - 2g_J = V - L + F_J.$$

Since $L = \frac{D+1}{2}V$, summing over all jackets we get

$$\sum_J F_J = (D-1)!F = -2\sum_J g_J + \frac{D!}{2}(2 + \frac{D-1}{2}V)$$

$$(D-1)!F - \frac{D!(D-1)V}{4} = D! - 2\omega$$

$$F = D + \frac{D(D-1)}{4}V - \frac{2}{D!}\omega$$

Choosing the $N^{-\frac{D(D-1)}{4}}$ scaling we get

$$A_G = |\lambda|^V N^{F - \frac{D(D-1)}{4}V} = |\lambda|^V N^{D - \frac{2\omega}{D!}}$$

Counting Faces with Jackets

Each face f_{ij} belongs to $(D-1)!$ jackets (the ones in which i and j are adjacent).

$$2 - 2g_J = V - L + F_J.$$

Since $L = \frac{D+1}{2}V$, summing over all jackets we get

$$\sum_J F_J = (D-1)!F = -2\sum_J g_J + \frac{D!}{2}(2 + \frac{D-1}{2}V)$$

$$(D-1)!F - \frac{D!(D-1)V}{4} = D! - 2\omega$$

$$F = D + \frac{D(D-1)}{4}V - \frac{2}{D!}\omega$$

Choosing the $N^{-\frac{D(D-1)}{4}}$ scaling we get

$$A_G = |\lambda|^V N^{F - \frac{D(D-1)}{4}V} = |\lambda|^V N^{D - \frac{2\omega}{D!}}$$

Counting Faces with Jackets

Each face f_{ij} belongs to $(D-1)!$ jackets (the ones in which i and j are adjacent).

$$2 - 2g_J = V - L + F_J.$$

Since $L = \frac{D+1}{2}V$, summing over all jackets we get

$$\sum_J F_J = (D-1)!F = -2\sum_J g_J + \frac{D!}{2}(2 + \frac{D-1}{2}V)$$

$$(D-1)!F - \frac{D!(D-1)V}{4} = D! - 2\omega$$

$$F = D + \frac{D(D-1)}{4}V - \frac{2}{D!}\omega$$

Choosing the $N^{-\frac{D(D-1)}{4}}$ scaling we get

$$A_G = |\lambda|^V N^{F - \frac{D(D-1)}{4}V} = |\lambda|^V N^{D - \frac{2\omega}{D!}}$$

Counting Faces with Jackets

Each face f_{ij} belongs to $(D-1)!$ jackets (the ones in which i and j are adjacent).

$$2 - 2g_J = V - L + F_J.$$

Since $L = \frac{D+1}{2}V$, summing over all jackets we get

$$\sum_J F_J = (D-1)!F = -2\sum_J g_J + \frac{D!}{2}(2 + \frac{D-1}{2}V)$$

$$(D-1)!F - \frac{D!(D-1)V}{4} = D! - 2\omega$$

$$F = D + \frac{D(D-1)}{4}V - \frac{2}{D!}\omega$$

Choosing the $N^{-\frac{D(D-1)}{4}}$ scaling we get

$$A_G = |\lambda|^V N^{F - \frac{D(D-1)}{4}V} = |\lambda|^V N^{D - \frac{2\omega}{D!}}$$

Counting Faces with Jackets

Each face f_{ij} belongs to $(D-1)!$ jackets (the ones in which i and j are adjacent).

$$2 - 2g_J = V - L + F_J.$$

Since $L = \frac{D+1}{2}V$, summing over all jackets we get

$$\sum_J F_J = (D-1)!F = -2\sum_J g_J + \frac{D!}{2}(2 + \frac{D-1}{2}V)$$

$$(D-1)!F - \frac{D!(D-1)V}{4} = D! - 2\omega$$

$$F = D + \frac{D(D-1)}{4}V - \frac{2}{D!}\omega$$

Choosing the $N^{-\frac{D(D-1)}{4}}$ scaling we get

$$A_G = |\lambda|^V N^{F - \frac{D(D-1)}{4}V} = |\lambda|^V N^{D - \frac{2\omega}{D!}}$$

Counting Faces with Jackets

Each face f_{ij} belongs to $(D-1)!$ jackets (the ones in which i and j are adjacent).

$$2 - 2g_J = V - L + F_J.$$

Since $L = \frac{D+1}{2}V$, summing over all jackets we get

$$\sum_J F_J = (D-1)!F = -2\sum_J g_J + \frac{D!}{2}(2 + \frac{D-1}{2}V)$$

$$(D-1)!F - \frac{D!(D-1)V}{4} = D! - 2\omega$$

$$F = D + \frac{D(D-1)}{4}V - \frac{2}{D!}\omega$$

Choosing the $N^{-\frac{D(D-1)}{4}}$ scaling we get

$$A_G = |\lambda|^V N^{F - \frac{D(D-1)}{4}V} = |\lambda|^V N^{D - \frac{2\omega}{D!}}$$

Melonic Graphs have Zero Degree

Recall that $F = D + \frac{D(D-1)}{4} V - \frac{2}{D!}\omega$ hence

$$\omega = 0 <=> F = D + \frac{D(D-1)}{4} V \qquad (A)$$

The elementary melon has $V = 2$ and $F = D(D+1)/2 = D + 2\frac{D(D-1)}{4}$.

By induction, since melonic insertion increases V by 2 and F by $\frac{D(D-1)}{2}$, any melonic graph has $F = D + \frac{D(D-1)}{4} V$ hence has $\omega = 0$, hence is a ZDG.

Melonic Graphs have Zero Degree

Recall that $F = D + \frac{D(D-1)}{4} V - \frac{2}{D!}\omega$ hence

$$\omega = 0 <=> F = D + \frac{D(D-1)}{4} V \qquad (A)$$

The elementary melon has $V = 2$ and $F = D(D+1)/2 = D + 2\frac{D(D-1)}{4}$.

By induction, since melonic insertion increases V by 2 and F by $\frac{D(D-1)}{2}$, any melonic graph has $F = D + \frac{D(D-1)}{4} V$ hence has $\omega = 0$, hence is a ZDG.

Melonic Graphs have Zero Degree

Recall that $F = D + \frac{D(D-1)}{4} V - \frac{2}{D!}\omega$ hence

$$\omega = 0 <=> F = D + \frac{D(D-1)}{4} V \qquad (A)$$

The elementary melon has $V = 2$ and $F = D(D+1)/2 = D + 2\frac{D(D-1)}{4}$.

By induction, since melonic insertion increases V by 2 and F by $\frac{D(D-1)}{2}$, any melonic graph has $F = D + \frac{D(D-1)}{4} V$ hence has $\omega = 0$, hence is a ZDG.

Zero Degree Graphs are melonic

Consider a ZDG. Call F_k the number of its faces of length $2k$. Recall that

$$F = \sum_{k \geq 1} F_k = D + \frac{D(D-1)}{4} V \qquad (A)$$

- Check by edge counting that

$$2F_1 + 4F_2 + \sum_{k \geq 3} 2kF_k = \frac{D(D+1)}{2} V \qquad (B)$$

- Compute $2A - B/2$ to prove that

$$F_1 = 2D + \sum_{k \geq 3} (k-2) F_k + \frac{D(D-3)}{4} V \geq 2D$$

  Conclude that vacuum (and also 2-point) ZDG's have faces of length 2.

- Conclude that any ZDG is a melon, hence $ZDG = \text{Melons}$ (Bonzom, Gurau, Riello, R, 2011, Witten 2016).

Zero Degree Graphs are melonic

Consider a ZDG. Call F_k the number of its faces of length $2k$. Recall that

$$F = \sum_{k \geq 1} F_k = D + \frac{D(D-1)}{4} V \qquad (A)$$

- Check by edge counting that

$$2F_1 + 4F_2 + \sum_{k \geq 3} 2kF_k = \frac{D(D+1)}{2} V \qquad (B)$$

- Compute $2A - B/2$ to prove that

$$F_1 = 2D + \sum_{k \geq 3} (k-2) F_k + \frac{D(D-3)}{4} V \geq 2D$$

  Conclude that vacuum (and also 2-point) ZDG's have faces of length 2.

- Conclude that any ZDG is a melon, hence $ZDG =$ Melons (Bonzom, Gurau, Riello, R, 2011, Witten 2016).

Zero Degree Graphs are melonic

Consider a ZDG. Call F_k the number of its faces of length $2k$. Recall that

$$F = \sum_{k\geq 1} F_k = D + \frac{D(D-1)}{4} V \qquad (A)$$

- Check by edge counting that

$$2F_1 + 4F_2 + \sum_{k\geq 3} 2kF_k = \frac{D(D+1)}{2} V \qquad (B)$$

- Compute $2A - B/2$ to prove that

$$F_1 = 2D + \sum_{k\geq 3} (k-2) F_k + \frac{D(D-3)}{4} V \geq 2D$$

Conclude that vacuum (and also 2-point) ZDG's have faces of length 2.

- Conclude that any ZDG is a melon, hence $ZDG =$ Melons (Bonzom, Gurau, Riello, R, 2011, Witten 2016).

Zero Degree Graphs are melonic

Consider a ZDG. Call F_k the number of its faces of length $2k$. Recall that

$$F = \sum_{k \geq 1} F_k = D + \frac{D(D-1)}{4} V \qquad (A)$$

- Check by edge counting that

$$2F_1 + 4F_2 + \sum_{k \geq 3} 2kF_k = \frac{D(D+1)}{2} V \qquad (B)$$

- Compute $2A - B/2$ to prove that

$$F_1 = 2D + \sum_{k \geq 3} (k-2) F_k + \frac{D(D-3)}{4} V \geq 2D$$

  Conclude that vacuum (and also 2-point) ZDG's have faces of length 2.

- Conclude that any ZDG is a melon, hence $ZDG =$ Melons (Bonzom, Gurau, Riello, R, 2011, Witten 2016).

Melons are Robust!

It is more difficult to count faces for symmetrized or antisymmetrized tensors... because the colors are no longer there to help.

Klebanov-Tarnopolosky: should melons also dominate at large N for symmetric traceless tensors?

Answer: Yes! (Carrozza et al, 2017- 2018) : melons dominate all rank-three irreducible representations of $O(N)$ and $Sp(N)$.

Conjecture: melons dominate all irreducible $O(N)$ representations at any finite rank ≥ 3.

Melons are Robust!

It is more difficult to count faces for symmetrized or antisymmetrized tensors... because the colors are no longer there to help.

Klebanov-Tarnopolosky: should melons also dominate at large N for symmetric traceless tensors?

Answer: Yes! (Carrozza et al, 2017- 2018) : melons dominate all rank-three irreducible representations of $O(N)$ and $Sp(N)$.

Conjecture: melons dominate all irreducible $O(N)$ representations at any finite rank ≥ 3.

Melons are Robust!

It is more difficult to count faces for symmetrized or antisymmetrized tensors... because the colors are no longer there to help.

Klebanov-Tarnopolosky: should melons also dominate at large N for symmetric traceless tensors?

Answer: Yes! (Carrozza et al, 2017- 2018) : melons dominate all rank-three irreducible representations of $O(N)$ and $Sp(N)$.

Conjecture: melons dominate all irreducible $O(N)$ representations at any finite rank ≥ 3.

Melons are Robust!

It is more difficult to count faces for symmetrized or antisymmetrized tensors... because the colors are no longer there to help.

Klebanov-Tarnopolosky: should melons also dominate at large N for symmetric traceless tensors?

Answer: Yes! (Carrozza et al, 2017- 2018) : melons dominate all rank-three irreducible representations of $O(N)$ and $Sp(N)$.

Conjecture: melons dominate all irreducible $O(N)$ representations at any finite rank ≥ 3.

Some Tensor Tracks Results (before 2016)

- Tensor models sum piecewise-linear quasi-manifolds with mild singularities
- Renormalizability of associated non-local field theories
- Asymptotic Freedom
- Constructive Aspects
- Numerical exploration of infrared critical points through FRG,

thanks to Benedetti, Ben Geloun, Bonzom, Carrozza, Dartois, Delepouve, Eichhorn, Erbin, Koslowski, Krajewski, Lahoche, Lionni, Oriti, Ryan, Samary, Tanasa, Toriumi, Vignes-Tourneret...

Some Tensor Tracks Results (before 2016)

- Tensor models sum piecewise-linear quasi-manifolds with mild singularities
- Renormalizability of associated non-local field theories
- Asymptotic Freedom
- Constructive Aspects
- Numerical exploration of infrared critical points through FRG,

thanks to Benedetti, Ben Geloun, Bonzom, Carrozza, Dartois, Delepouve, Eichhorn, Erbin, Koslowski, Krajewski, Lahoche, Lionni, Oriti, Ryan, Samary, Tanasa, Toriumi, Vignes-Tourneret...

Some Tensor Tracks Results (before 2016)

- Tensor models sum piecewise-linear quasi-manifolds with mild singularities
- Renormalizability of associated non-local field theories
- Asymptotic Freedom
- Constructive Aspects
- Numerical exploration of infrared critical points through FRG,

thanks to Benedetti, Ben Geloun, Bonzom, Carrozza, Dartois, Delepouve, Eichhorn, Erbin, Koslowski, Krajewski, Lahoche, Lionni, Oriti, Ryan, Samary, Tanasa, Toriumi, Vignes-Tourneret...

Some Tensor Tracks Results (before 2016)

- Tensor models sum piecewise-linear quasi-manifolds with mild singularities
- Renormalizability of associated non-local field theories
- Asymptotic Freedom
- Constructive Aspects
- Numerical exploration of infrared critical points through FRG,

thanks to Benedetti, Ben Geloun, Bonzom, Carrozza, Dartois, Delepouve, Eichhorn, Erbin, Koslowski, Krajewski, Lahoche, Lionni, Oriti, Ryan, Samary, Tanasa, Toriumi, Vignes-Tourneret...

Some Tensor Tracks Results (before 2016)

- Tensor models sum piecewise-linear quasi-manifolds with mild singularities
- Renormalizability of associated non-local field theories
- Asymptotic Freedom
- Constructive Aspects
- Numerical exploration of infrared critical points through FRG,

thanks to Benedetti, Ben Geloun, Bonzom, Carrozza, Dartois, Delepouve, Eichhorn, Erbin, Koslowski, Krajewski, Lahoche, Lionni, Oriti, Ryan, Samary, Tanasa, Toriumi, Vignes-Tourneret...

Some Tensor Tracks Results (before 2016)

- Tensor models sum piecewise-linear quasi-manifolds with mild singularities
- Renormalizability of associated non-local field theories
- Asymptotic Freedom
- Constructive Aspects
- Numerical exploration of infrared critical points through FRG,

thanks to Benedetti, Ben Geloun, Bonzom, Carrozza, Dartois, Delepouve, Eichhorn, Erbin, Koslowski, Krajewski, Lahoche, Lionni, Oriti, Ryan, Samary, Tanasa, Toriumi, Vignes-Tourneret...

Some Tensor Tracks Results (before 2016)

- Tensor models sum piecewise-linear quasi-manifolds with mild singularities
- Renormalizability of associated non-local field theories
- Asymptotic Freedom
- Constructive Aspects
- Numerical exploration of infrared critical points through FRG,

thanks to Benedetti, Ben Geloun, Bonzom, Carrozza, Dartois, Delepouve, Eichhorn, Erbin, Koslowski, Krajewski, Lahoche, Lionni, Oriti, Ryan, Samary, Tanasa, Toriumi, Vignes-Tourneret...

Critical Melons

- At the melonic critical point $\lambda = \lambda_c$ the melonic series diverge. The corresponding continuous random space for the graph distance is the Aldous tree (Gurau and Ryan 2013)
- There is a double scaling limit but it resums few graphs. For $d \leq 6$ there are indications of a triple scaling limit and a link with topological recursion (Dartois's work)
- There exist enhanced tensor models which interpolate between the Aldous tree and the brownian sphere. At the transition point in between lies a baby-universe phase (Bonzom, Delepouve, Lionni, R...).
- I see the tensor Aldous phase not as an end but as a starting point for further analysis. It would be good if eg a phase of the "random volume" type found by the CDT simulations could be obtained as a critical point of a tensorial quantum field theory.

Critical Melons

- At the melonic critical point $\lambda = \lambda_c$ the melonic series diverge. The corresponding continuous random space for the graph distance is the Aldous tree (Gurau and Ryan 2013)
- There is a double scaling limit but it resums few graphs. For $d \leq 6$ there are indications of a triple scaling limit and a link with topological recursion (Dartois's work)
- There exist enhanced tensor models which interpolate between the Aldous tree and the brownian sphere. At the transition point in between lies a baby-universe phase (Bonzom, Delepouve, Lionni, R...).
- I see the tensor Aldous phase not as an end but as a starting point for further analysis. It would be good if eg a phase of the "random volume" type found by the CDT simulations could be obtained as a critical point of a tensorial quantum field theory.

Critical Melons

- At the melonic critical point $\lambda = \lambda_c$ the melonic series diverge. The corresponding continuous random space for the graph distance is the Aldous tree (Gurau and Ryan 2013)
- There is a double scaling limit but it resums few graphs. For $d \leq 6$ there are indications of a triple scaling limit and a link with topological recursion (Dartois's work)
- There exist enhanced tensor models which interpolate between the Aldous tree and the brownian sphere. At the transition point in between lies a baby-universe phase (Bonzom, Delepouve, Lionni, R...).
- I see the tensor Aldous phase not as an end but as a starting point for further analysis. It would be good if eg a phase of the "random volume" type found by the CDT simulations could be obtained as a critical point of a tensorial quantum field theory.

Critical Melons

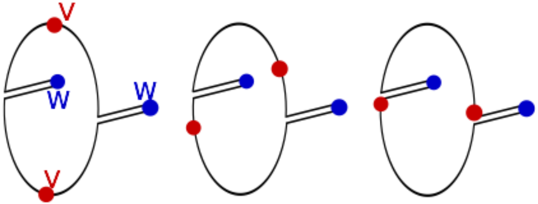
- At the melonic critical point $\lambda = \lambda_c$ the melonic series diverge. The corresponding continuous random space for the graph distance is the Aldous tree (Gurau and Ryan 2013)
- There is a double scaling limit but it resums few graphs. For $d \leq 6$ there are indications of a triple scaling limit and a link with topological recursion (Dartois's work)
- There exist enhanced tensor models which interpolate between the Aldous tree and the brownian sphere. At the transition point in between lies a baby-universe phase (Bonzom, Delepouve, Lionni, R...).
- I see the tensor Aldous phase not as an end but as a starting point for further analysis. It would be good if eg a phase of the "random volume" type found by the CDT simulations could be obtained as a critical point of a tensorial quantum field theory.

Critical Melons

- At the melonic critical point $\lambda = \lambda_c$ the melonic series diverge. The corresponding continuous random space for the graph distance is the Aldous tree (Gurau and Ryan 2013)
- There is a double scaling limit but it resums few graphs. For $d \leq 6$ there are indications of a triple scaling limit and a link with topological recursion (Dartois's work)
- There exist enhanced tensor models which interpolate between the Aldous tree and the brownian sphere. At the transition point in between lies a baby-universe phase (Bonzom, Delepouve, Lionni, R...).
- I see the tensor Aldous phase not as an end but as a starting point for further analysis. It would be good if eg a phase of the “random volume” type found by the CDT simulations could be obtained as a critical point of a tensorial quantum field theory.

A bound on chaos

In March 2015 Maldacena, Shenker and Stanford wrote that in any quantum system at temperature T the Lyapunov exponent for transient chaos in a four point correlator maximally spaced on the thermal circle

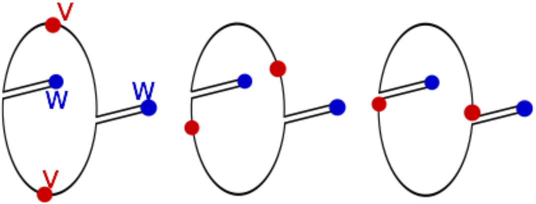


$$F(t) = \mathrm{Tr}r[yVyW(t)yVyW(t)], \quad y := Z^{-1/4}e^{-\beta H/4},$$

is bounded by $\lambda_L \leq 2\pi T/\hbar$ under very general assumptions (analyticity in a strip of width $\beta/2$ in complex time and reasonable decay at infinity).

A bound on chaos

In March 2015 Maldacena, Shenker and Stanford wrote that in any quantum system at temperature T the Lyapunov exponent for transient chaos in a four point correlator maximally spaced on the thermal circle

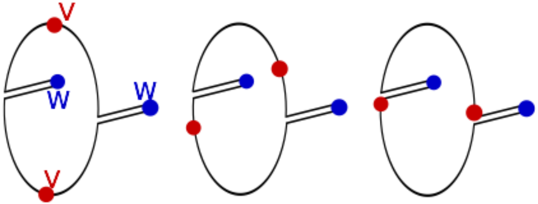


$$F(t) = \mathrm{Tr}r[yVyW(t)yVyW(t)], \quad y := Z^{-1/4}e^{-\beta H/4},$$

is bounded by $\lambda_L \leq 2\pi T/\hbar$ under very general assumptions (analyticity in a strip of width $\beta/2$ in complex time and reasonable decay at infinity).

A bound on chaos

In March 2015 Maldacena, Shenker and Stanford wrote that in any quantum system at temperature T the Lyapunov exponent for transient chaos in a four point correlator maximally spaced on the thermal circle

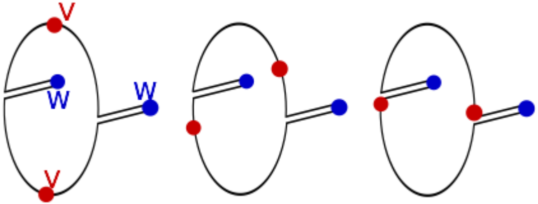


$$F(t) = \mathrm{Tr} r[yVyW(t)yVyW(t)], \quad y := Z^{-1/4} e^{-\beta H/4},$$

is bounded by $\lambda_L \leq 2\pi T/\hbar$ under very general assumptions (analyticity in a strip of width $\beta/2$ in complex time and reasonable decay at infinity).

A bound on chaos

In March 2015 Maldacena, Shenker and Stanford wrote that in any quantum system at temperature T the Lyapunov exponent for transient chaos in a four point correlator maximally spaced on the thermal circle



$$F(t) = \mathrm{Tr}r[yVyW(t)yVyW(t)], \quad y := Z^{-1/4}e^{-\beta H/4},$$

is bounded by $\lambda_L \leq 2\pi T/\hbar$ under very general assumptions (analyticity in a strip of width $\beta/2$ in complex time and reasonable decay at infinity).

The MSS bound

More precisely they found that

$$F \simeq (a - \frac{b}{N^2} e^{\lambda_L t})^{-b}, t_d < t < t_s$$

and that $\lambda_L \leq 2\pi T/\hbar$.

They argued convincingly that saturation of this bound is a strong indication of the presence of quantum gravity.

The MSS bound

More precisely they found that

$$F \simeq (a - \frac{b}{N^2} e^{\lambda_L t})^{-b}, t_d < t < t_s$$

and that $\lambda_L \leq 2\pi T/\hbar$.

They argued convincingly that saturation of this bound is a strong indication of the presence of quantum gravity.

The Sachdev-Ye-Kitaev Model

In 2015 Kitaev found a very simple quasi-conformal one dimensional quantum mechanics model which saturates the MSS bound, indicating the surprising presence of a gravitational dual.

The action is

$$I = \int dt \left(\frac{i}{2} \sum_i \psi_i \frac{d}{dt} \psi_i - i^{q/2} \sum_{1 \le i_1 < \cdots < i_q \le N} J_{i_1, \cdots, i_q} \psi_{i_1} \cdots \psi_{i_q} \right) \tag{3.2}$$

with J a quenched iid random tensor ($< J_I J_{I'} >= \delta_{II'} J^2 (q-1)! N^{-(q-1)}$), and ψ an N-vector Majorana Fermion.

I would like to call such theories tensor-vectors (p-spin models in disordered systems, spiked tensor model in data analysis, isotropic models in the theory of Gaussian processes all fall in that category).

The Sachdev-Ye-Kitaev Model

In 2015 Kitaev found a very simple quasi-conformal one dimensional quantum mechanics model which saturates the MSS bound, indicating the surprising presence of a gravitational dual.

The action is

$$I = \int dt \left(\frac{i}{2} \sum_i \psi_i \frac{d}{dt} \psi_i - i^{q/2} \sum_{1 \le i_1 < \cdots < i_q \le N} J_{i_1, \cdots, i_q} \psi_{i_1} \cdots \psi_{i_q} \right) \tag{3.2}$$

with J a quenched iid random tensor ($< J_I J_{I'} >= \delta_{II'} J^2 (q-1)! N^{-(q-1)}$), and ψ an N-vector Majorana Fermion.

I would like to call such theories tensor-vectors (p-spin models in disordered systems, spiked tensor model in data analysis, isotropic models in the theory of Gaussian processes all fall in that category).

The Sachdev-Ye-Kitaev Model

In 2015 Kitaev found a very simple quasi-conformal one dimensional quantum mechanics model which saturates the MSS bound, indicating the surprising presence of a gravitational dual.

The action is

$$I = \int dt \left(\frac{i}{2} \sum_i \psi_i \frac{d}{dt} \psi_i - i^{q/2} \sum_{1 \le i_1 < \cdots < i_q \le N} J_{i_1, \cdots, i_q} \psi_{i_1} \cdots \psi_{i_q} \right) \tag{3.2}$$

with J a quenched iid random tensor ($< J_I J_{I'} >= \delta_{II'} J^2 (q-1)! N^{-(q-1)}$), and ψ an N-vector Majorana Fermion.

I would like to call such theories tensor-vectors (p-spin models in disordered systems, spiked tensor model in data analysis, isotropic models in the theory of Gaussian processes all fall in that category).

The Sachdev-Ye-Kitaev Model

In 2015 Kitaev found a very simple quasi-conformal one dimensional quantum mechanics model which saturates the MSS bound, indicating the surprising presence of a gravitational dual.

The action is

$$I = \int dt \left(\frac{i}{2} \sum_i \psi_i \frac{d}{dt} \psi_i - i^{q/2} \sum_{1 \le i_1 < \cdots < i_q \le N} J_{i_1, \cdots, i_q} \psi_{i_1} \cdots \psi_{i_q} \right) \tag{3.2}$$

with J a quenched iid random tensor ($< J_I J_{I'} >= \delta_{II'} J^2 (q-1)! N^{-(q-1)}$), and ψ an N-vector Majorana Fermion.

I would like to call such theories tensor-vectors (p-spin models in disordered systems, spiked tensor model in data analysis, isotropic models in the theory of Gaussian processes all fall in that category).

The Sachdev-Ye-Kitaev Model

In 2015 Kitaev found a very simple quasi-conformal one dimensional quantum mechanics model which saturates the MSS bound, indicating the surprising presence of a gravitational dual.

The action is

$$I = \int dt \Big(\frac{i}{2} \sum_i \psi_i \frac{d}{dt} \psi_i - i^{q/2} \sum_{1 \le i_1 < \cdots < i_q \le N} J_{i_1, \cdots, i_q} \psi_{i_1} \cdots \psi_{i_q} \Big) \tag{3.2}$$

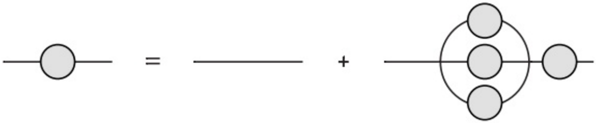
with J a quenched iid random tensor ($< J_I J_{I'} >= \delta_{II'} J^2 (q-1)! N^{-(q-1)}$), and ψ an N-vector Majorana Fermion.

I would like to call such theories tensor-vectors (p-spin models in disordered systems, spiked tensor model in data analysis, isotropic models in the theory of Gaussian processes all fall in that category).

The SYK Model

This model is solvable as $N \to \infty$, being approximately conformal and reparametrization invariant in the infra-red limit.

The reason it can be solved in the limit $N \to \infty$ is because the leading Feynman graphs are melons.



For instance the two point function in that limit reads

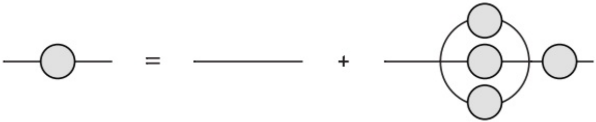
$$G(\tau) = b_q \Big[\frac{\pi}{\beta \sin(\pi\tau/\beta)}\Big]^{2/q} \mathrm{sgn}\tau.$$

The 4 point function computation is more subtle but the result shows saturation of the MSS bound. This amounts to a $NAdS_2/NCFT_1$ holographic correspondence.

The SYK Model

This model is solvable as $N \to \infty$, being approximately conformal and reparametrization invariant in the infra-red limit.

The reason it can be solved in the limit $N \to \infty$ is because the leading Feynman graphs are melons.



For instance the two point function in that limit reads

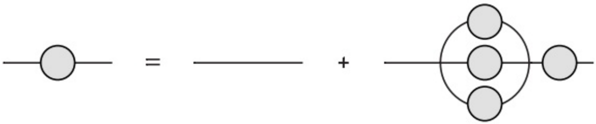
$$G(\tau) = b_q[\frac{\pi}{\beta \sin(\pi\tau/\beta)}]^{2/q} \mathrm{sgn}\tau.$$

The 4 point function computation is more subtle but the result shows saturation of the MSS bound. This amounts to a $NAdS_2/NCFT_1$ holographic correspondence.

The SYK Model

This model is solvable as $N \to \infty$, being approximately conformal and reparametrization invariant in the infra-red limit.

The reason it can be solved in the limit $N \to \infty$ is because the leading Feynman graphs are melons.



For instance the two point function in that limit reads

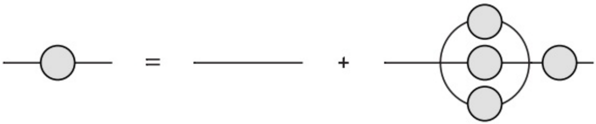
$$G(\tau) = b_q[\frac{\pi}{\beta \sin(\pi\tau/\beta)}]^{2/q}\mathrm{sgn}\tau.$$

The 4 point function computation is more subtle but the result shows saturation of the MSS bound. This amounts to a $NAdS_2/NCFT_1$ holographic correspondence.

The SYK Model

This model is solvable as $N \to \infty$, being approximately conformal and reparametrization invariant in the infra-red limit.

The reason it can be solved in the limit $N \to \infty$ is because the leading Feynman graphs are melons.



For instance the two point function in that limit reads

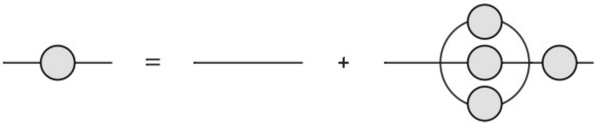
$$G(\tau) = b_q [\frac{\pi}{\beta \sin(\pi\tau/\beta)}]^{2/q} \mathrm{sgn}\tau.$$

The 4 point function computation is more subtle but the result shows saturation of the MSS bound. This amounts to a $NAdS_2/NCFT_1$ holographic correspondence.

The SYK Model

This model is solvable as $N \to \infty$, being approximately conformal and reparametrization invariant in the infra-red limit.

The reason it can be solved in the limit $N \to \infty$ is because the leading Feynman graphs are melons.



For instance the two point function in that limit reads

$$G(\tau) = b_q[\frac{\pi}{\beta \sin(\pi\tau/\beta)}]^{2/q} \mathrm{sgn}\tau.$$

The 4 point function computation is more subtle but the result shows saturation of the MSS bound. This amounts to a $NAdS_2/NCFT_1$ holographic correspondence.

Gurau-Witten Models

Late in 2016 E. Witten remarked the link between the SYK model and random tensors.

He proposed a modification to eliminate the quenched disorder with action

$$I = \int dt \left(\frac{i}{2} \sum_i \psi_i \frac{d}{dt} \psi_i - i^{q/2} j \psi_0 \psi_1 \cdots \psi_D \right) \tag{3.3}$$

where ψ's are $D+1$ fermionic tensors and the pattern of index contraction is exactly the one of Gurau's initial colored tensor model, hence this new model is now called the Gurau-Witten model.

An uncolored, i.e. single tensor model with similar properties was soon developped by Klebanov and Tarnopolsky, based on the three dimensional $O(N)$ tensor model of Carrozza and Tanasa (2015).

Gurau-Witten Models

Late in 2016 E. Witten remarked the link between the SYK model and random tensors.

He proposed a modification to eliminate the quenched disorder with action

$$I = \int dt \left(\frac{i}{2} \sum_i \psi_i \frac{d}{dt} \psi_i - i^{q/2} j \psi_0 \psi_1 \cdots \psi_D \right) \tag{3.3}$$

where ψ's are $D+1$ fermionic tensors and the pattern of index contraction is exactly the one of Gurau's initial colored tensor model, hence this new model is now called the Gurau-Witten model.

An uncolored, i.e. single tensor model with similar properties was soon developped by Klebanov and Tarnopolsky, based on the three dimensional $O(N)$ tensor model of Carrozza and Tanasa (2015).

Gurau-Witten Models

Late in 2016 E. Witten remarked the link between the SYK model and random tensors.

He proposed a modification to eliminate the quenched disorder with action

$$I = \int dt \left(\frac{i}{2} \sum_i \psi_i \frac{d}{dt} \psi_i - i^{q/2} j \psi_0 \psi_1 \cdots \psi_D \right) \tag{3.3}$$

where ψ's are $D+1$ fermionic tensors and the pattern of index contraction is exactly the one of Gurau's initial colored tensor model, hence this new model is now called the Gurau-Witten model.

An uncolored, i.e. single tensor model with similar properties was soon developped by Klebanov and Tarnopolsky, based on the three dimensional $O(N)$ tensor model of Carrozza and Tanasa (2015).

Gurau-Witten Models

Late in 2016 E. Witten remarked the link between the SYK model and random tensors.

He proposed a modification to eliminate the quenched disorder with action

$$I = \int dt \left(\frac{i}{2} \sum_i \psi_i \frac{d}{dt} \psi_i - i^{q/2} j \psi_0 \psi_1 \cdots \psi_D \right) \tag{3.3}$$

where ψ's are $D+1$ fermionic tensors and the pattern of index contraction is exactly the one of Gurau's initial colored tensor model, hence this new model is now called the Gurau-Witten model.

An uncolored, i.e. single tensor model with similar properties was soon developped by Klebanov and Tarnopolsky, based on the three dimensional $O(N)$ tensor model of Carrozza and Tanasa (2015).

Gurau-Witten Models

Late in 2016 E. Witten remarked the link between the SYK model and random tensors.

He proposed a modification to eliminate the quenched disorder with action

$$I = \int dt \left(\frac{i}{2} \sum_i \psi_i \frac{d}{dt} \psi_i - i^{q/2} j \psi_0 \psi_1 \cdots \psi_D \right) \tag{3.3}$$

where ψ's are $D+1$ fermionic tensors and the pattern of index contraction is exactly the one of Gurau's initial colored tensor model, hence this new model is now called the Gurau-Witten model.

An uncolored, i.e. single tensor model with similar properties was soon developped by Klebanov and Tarnopolsky, based on the three dimensional $O(N)$ tensor model of Carrozza and Tanasa (2015).

Random Geometry and Holography

- Tensor models à la Gurau-Witten are true quantum tensor theories, not quenched averages of tensor-vectors theories like SYK.
- To understand the physics of the gravity side in this $NCFT_1/NAdS_2$ correspondence is a hot current topic.
- A main issue for the future is in my opinion to understand the link between the random geometric and the holographic aspects of random tensors.

Random Holography?

Random Geometry and Holography

- Tensor models à la Gurau-Witten are true quantum tensor theories, not quenched averages of tensor-vectors theories like SYK.
- To understand the physics of the gravity side in this $NCFT_1/NAdS_2$ correspondence is a hot current topic.
- A main issue for the future is in my opinion to understand the link between the random geometric and the holographic aspects of random tensors.

Random Holography?

Random Geometry and Holography

- Tensor models à la Gurau-Witten are true quantum tensor theories, not quenched averages of tensor-vectors theories like SYK.
- To understand the physics of the gravity side in this $NCFT_1/NAdS_2$ correspondence is a hot current topic.
- A main issue for the future is in my opinion to understand the link between the random geometric and the holographic aspects of random tensors.

Random Holography?

Random Geometry and Holography

- Tensor models à la Gurau-Witten are true quantum tensor theories, not quenched averages of tensor-vectors theories like SYK.
- To understand the physics of the gravity side in this $NCFT_1/NAdS_2$ correspondence is a hot current topic.
- A main issue for the future is in my opinion to understand the link between the random geometric and the holographic aspects of random tensors.

Random Holography?

Random Geometry and Holography

- Tensor models à la Gurau-Witten are true quantum tensor theories, not quenched averages of tensor-vectors theories like SYK.
- To understand the physics of the gravity side in this $NCFT_1/NAdS_2$ correspondence is a hot current topic.
- A main issue for the future is in my opinion to understand the link between the random geometric and the holographic aspects of random tensors.

Random Holography?

An Application: Melonic Turbulence

Dartois-Evnin-Lionni-R-Valette considered recently (arXiv:1810.01848) a specific resonant non-linear equation, namely

$$i\,\frac{d\alpha_j}{dt}(t) = \sum_{\substack{j',k,k'=0 \\ j+j'=k+k'}}^{\infty} C_{jj'kk'}\bar{\alpha}_{j'}(t)\alpha_k(t)\alpha_{k'}(t)$$

on the (infinite) collection of modes α_n, $n \in \mathbb{N}$.

Such equations naturally emerge in many weakly non-linear PDEs with highly resonant linearized spectra (cubic Szegö, non-linear Schrödinger, Bose-Einstein condensates in harmonic trap, non-linear dynamics in AdS space...)

An Application: Melonic Turbulence

Dartois-Evnin-Lionni-R-Valette considered recently (arXiv:1810.01848) a specific resonant non-linear equation, namely

$$i \frac{d\alpha_j}{dt}(t) = \sum_{\substack{j',k,k'=0 \\ j+j'=k+k'}}^{\infty} C_{jj'kk'} \bar{\alpha}_{j'}(t)\alpha_k(t)\alpha_{k'}(t)$$

on the (infinite) collection of modes α_n, $n \in \mathbb{N}$.

Such equations naturally emerge in many weakly non-linear PDEs with highly resonant linearized spectra (cubic Szegö, non-linear Schrödinger, Bose-Einstein condensates in harmonic trap, non-linear dynamics in AdS space...)

A Typical Example

$d = 1$ non-linear Schrödinger equation in harmonic trap

$$i\frac{\partial \Psi}{\partial t} = \frac{1}{2}\left(-\frac{\partial^2}{\partial x^2} + x^2\right)\Psi + g|\Psi|^2\Psi,$$

The linearized problem ($g = 0$) is a Schrödinger equation with solution

$$\Psi = \sum_{n=0}^{\infty} \alpha_n \psi_n(x) e^{-iE_n t}, \qquad E_n = n + \frac{1}{2}, \qquad \frac{1}{2}\left(-\frac{\partial^2}{\partial x^2} + x^2\right)\psi_n = E_n \psi_n,$$

with constant α_n. In the weakly non-linear regime $g \ll 1$, α_n acquire slow drifts. Substituting and projecting on $\psi_k(x)$ yields

$$i\frac{d}{dt}\alpha_j(t) = g \sum_{j',k,k'=\emptyset}^{\infty} C_{jj'kk'}\bar{\alpha}_{j'}(t)\alpha_k(t)\alpha_{k'}(t)\, e^{i(E_j+E_{j'}-E_k-E_{k'})t},$$

where $C_{jj'kk'} = \int dx\, \psi_j \psi_{j'} \psi_k \psi_{k'}$. Discarding fast oscillations leads to our equation with resonant condition $E_j + E_{j'} - E_k - E_{k'} \equiv j + j' - k - k' = 0$.

Any classical non linear flow $\dot{q}_i = \lambda T_{ijk\ldots} q_j q_k \ldots$ can be solved in power series in t as a sum over trees (Poincaré-Linstedt...).

We Gaussian-average over initial conditions which excite many modes

$$\langle \alpha_j(0) \bar{\alpha}_{j'}(0) \rangle_\alpha = \frac{\delta_{jj'}}{N} \chi_N(j),$$

where e.g. $\chi_N(j) = 1$ if $j < N$ and $\chi_N(j) = 1$ if $j \geq N$, and over the non-linear couplings C.

The averaged Poincaré-Linstedt series becomes a Feynman graphs series. In turbulence this apporoach goes back to Kraichnan (Direct Interaction Approximation) and in disordered systems goes under the name of Mode Coupling Approximation (Bouchaud-Cugliandolo 1996).

Any classical non linear flow $\dot{q}_i = \lambda T_{ijk\ldots} q_j q_k \ldots$ can be solved in power series in t as a sum over trees (Poincaré-Linstedt...).

We Gaussian-average over initial conditions which excite many modes

$$\langle \alpha_j(0) \bar{\alpha}_{j'}(0) \rangle_\alpha = \frac{\delta_{jj'}}{N} \chi_N(j),$$

where e.g. $\chi_N(j) = 1$ if $j < N$ and $\chi_N(j) = 1$ if $j \geq N$, and over the non-linear couplings C.

The averaged Poincaré-Linstedt series becomes a Feynman graphs series. In turbulence this apporoach goes back to Kraichnan (Direct Interaction Approximation) and in disordered systems goes under the name of Mode Coupling Approximation (Bouchaud-Cugliandolo 1996).

Any classical non linear flow $\dot{q}_i = \lambda T_{ijk\dots} q_j q_k \dots$ can be solved in power series in t as a sum over trees (Poincaré-Linstedt...).

We Gaussian-average over initial conditions which excite many modes

$$\langle \alpha_j(0) \bar{\alpha}_{j'}(0) \rangle_\alpha = \frac{\delta_{jj'}}{N} \chi_N(j),$$

where e.g. $\chi_N(j) = 1$ if $j < N$ and $\chi_N(j) = 1$ if $j \geq N$, and over the non-linear couplings C.

The averaged Poincaré-Linstedt series becomes a Feynman graphs series. In turbulence this apporoach goes back to Kraichnan (Direct Interaction Approximation) and in disordered systems goes under the name of Mode Coupling Approximation (Bouchaud-Cugliandolo 1996).

Any classical non linear flow $\dot{q}_i = \lambda T_{ijk\ldots} q_j q_k \ldots$ can be solved in power series in t as a sum over trees (Poincaré-Linstedt...).

We Gaussian-average over initial conditions which excite many modes

$$\langle \alpha_j(0) \bar{\alpha}_{j'}(0) \rangle_\alpha = \frac{\delta_{jj'}}{N} \chi_N(j),$$

where e.g. $\chi_N(j) = 1$ if $j < N$ and $\chi_N(j) = 1$ if $j \geq N$, and over the non-linear couplings C.

The averaged Poincaré-Linstedt series becomes a Feynman graphs series. In turbulence this apporoach goes back to Kraichnan (Direct Interaction Approximation) and in disordered systems goes under the name of Mode Coupling Approximation (Bouchaud-Cugliandolo 1996).

Our Results

Consider the averaged Sobolev norms

$$S_\gamma(t) =< \sum_{r\geq 0} r^\gamma \bar{\alpha}_r(t)\alpha_r(t) >_{\alpha(0),C}$$

Theorem 1 *The dominant graphs as $N \to \infty$ for $S_\gamma(t)$ are exactly the melonic graphs. The corresponding approximation $S_\gamma^{melo}(t)$ is an analytic function of time in a disk $|t| < \rho$ of finite radius $\rho > 0$.*

Theorem 2 *For any $\gamma > 1$ there exists a constant δ such that $S_\gamma^{melo}(t)$ grows monotonically in time for $t \in [0, \delta]$.*

It means that *in the melonic approximation*, energy spreads at least for a while from the low modes to the higher modes, as expected in a turbulent cascade. We call this phenomenon *melonic turbulence*,

Our Results

Consider the averaged Sobolev norms

$$S_\gamma(t) = < \sum_{r \geq 0} r^\gamma \bar{\alpha}_r(t) \alpha_r(t) >_{\alpha(0),C}$$

Theorem 1 *The dominant graphs as $N \to \infty$ for $S_\gamma(t)$ are exactly the melonic graphs. The corresponding approximation $S_\gamma^{melo}(t)$ is an analytic function of time in a disk $|t| < \rho$ of finite radius $\rho > 0$.*

Theorem 2 *For any $\gamma > 1$ there exists a constant δ such that $S_\gamma^{melo}(t)$ grows monotonically in time for $t \in [0, \delta]$.*

It means that *in the melonic approximation*, energy spreads at least for a while from the low modes to the higher modes, as expected in a turbulent cascade. We call this phenomenon *melonic turbulence*,

Our Results

Consider the averaged Sobolev norms

$$S_\gamma(t) = < \sum_{r \geq 0} r^\gamma \bar{\alpha}_r(t) \alpha_r(t) >_{\alpha(0),C}$$

Theorem 1 *The dominant graphs as $N \to \infty$ for $S_\gamma(t)$ are exactly the melonic graphs. The corresponding approximation $S_\gamma^{melo}(t)$ is an analytic function of time in a disk $|t| < \rho$ of finite radius $\rho > 0$.*

Theorem 2 *For any $\gamma > 1$ there exists a constant δ such that $S_\gamma^{melo}(t)$ grows monotonically in time for $t \in [0, \delta]$.*

It means that *in the melonic approximation*, energy spreads at least for a while from the low modes to the higher modes, as expected in a turbulent cascade. We call this phenomenon *melonic turbulence*,

Our Results

Consider the averaged Sobolev norms

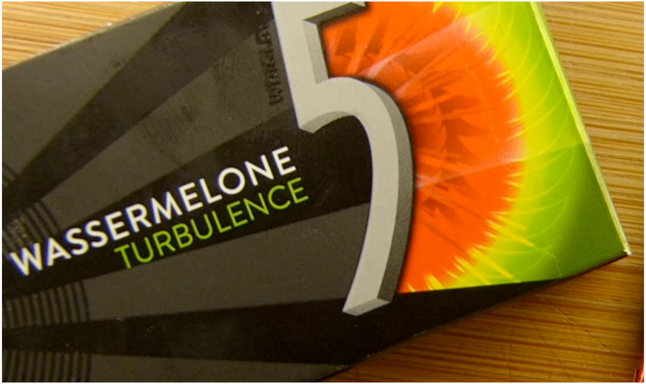
$$S_\gamma(t) = < \sum_{r \geq 0} r^\gamma \bar{\alpha}_r(t) \alpha_r(t) >_{\alpha(0),C}$$

Theorem 1 *The dominant graphs as $N \to \infty$ for $S_\gamma(t)$ are exactly the melonic graphs. The corresponding approximation $S_\gamma^{melo}(t)$ is an analytic function of time in a disk $|t| < \rho$ of finite radius $\rho > 0$.*

Theorem 2 *For any $\gamma > 1$ there exists a constant δ such that $S_\gamma^{melo}(t)$ grows monotonically in time for $t \in [0, \delta]$.*

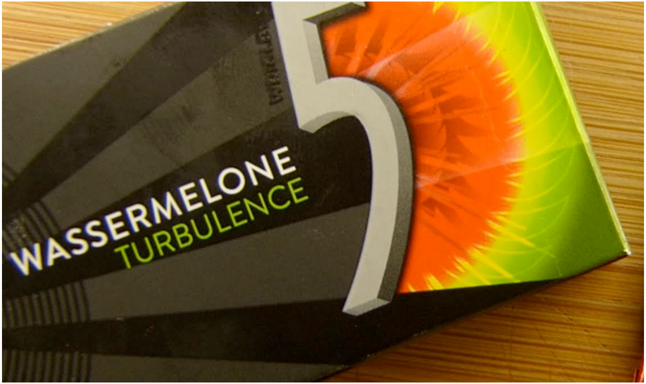
It means that *in the melonic approximation*, energy spreads at least for a while from the low modes to the higher modes, as expected in a turbulent cascade. We call this phenomenon *melonic turbulence*,

WasserMelone



although we may have copyright problems...

WasserMelone



although we may have copyright problems...

Conclusion

A theory of some things...

Conclusion

A theory of some things...

Conclusion

with more surprises to come?

Conclusion

with more surprises to come?