Janossy densities of chiral random matrix models and QCD Dirac spectra

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Janossy densities [§2]

of chiral random matrix models

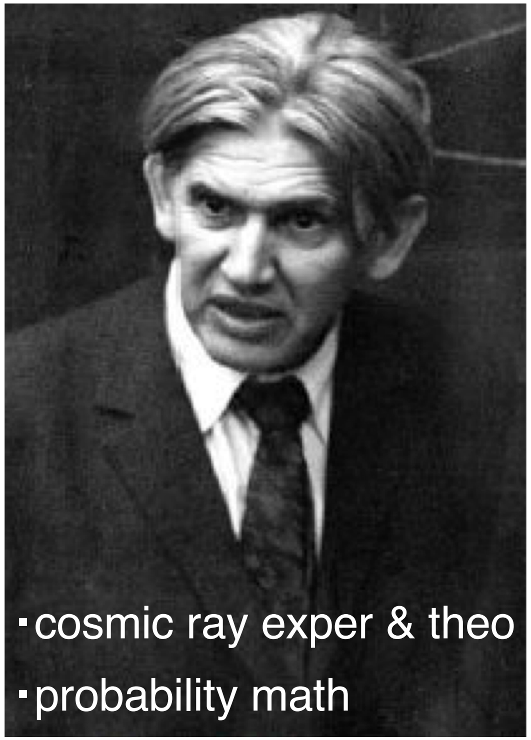
§4 of WTC candidates

and QCD Dirac spectra

§1 Introduction

§3 Ordered EV statistics

§5 Summary



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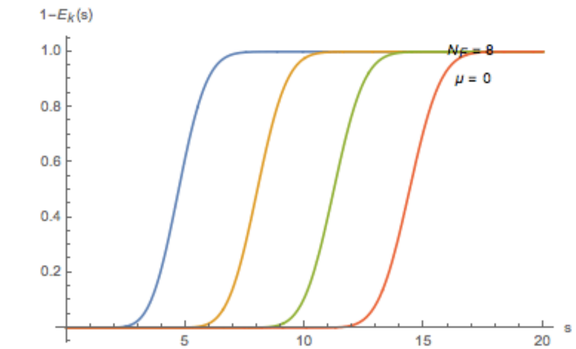
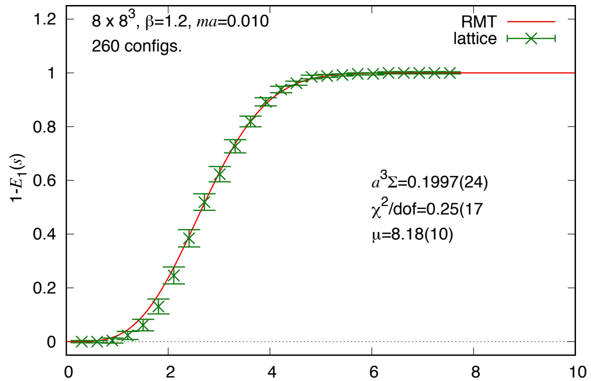
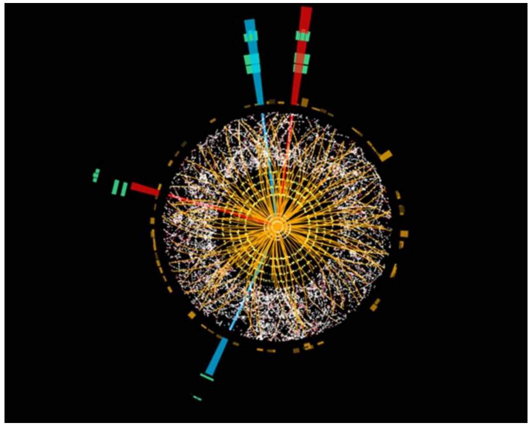


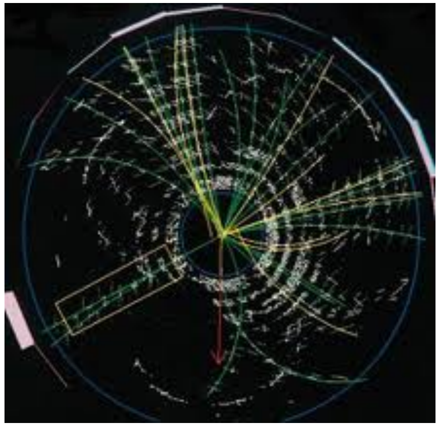


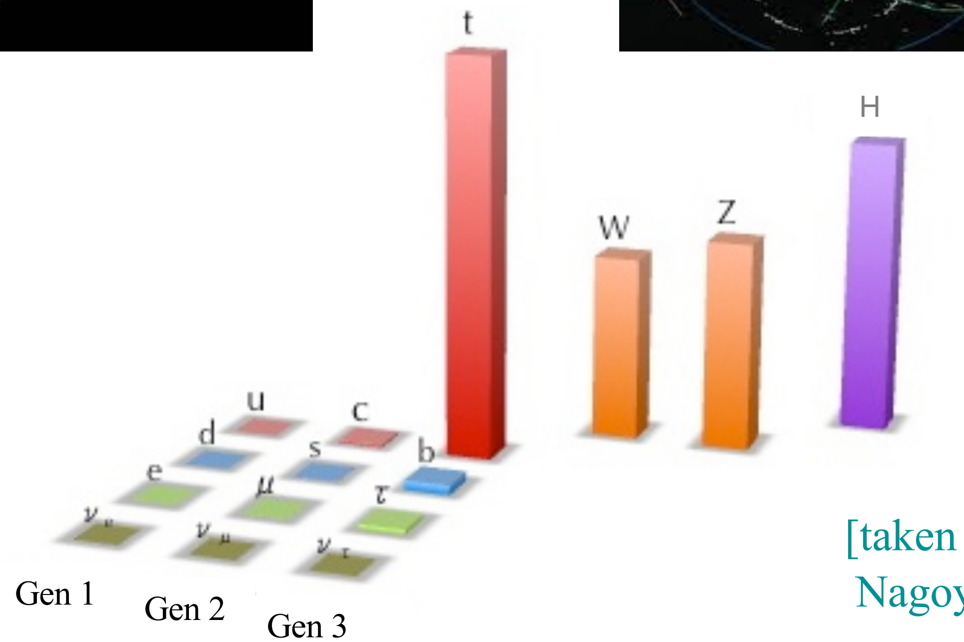
Hierarchy problem of SM

Higgs boson M=125GeV ≈ Top quark m=173GeV

LHC@CERN 2012; 2013 Tevatron@FNAL 1995; 2008





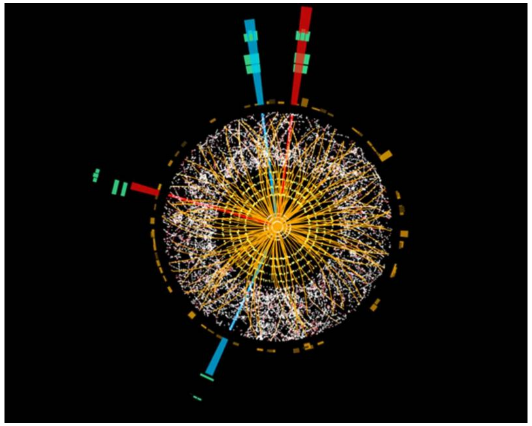


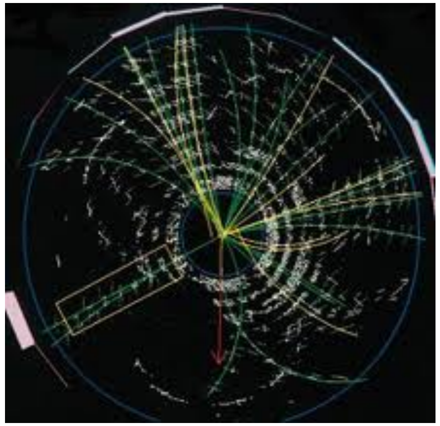
[taken from
Nagoya KMI HP]

Hierarchy problem of SM

Higgs boson M=125GeV ≈ Top quark m=173GeV

LHC@CERN 2012;  2013 Tevatron@FNAL 1995;  2008



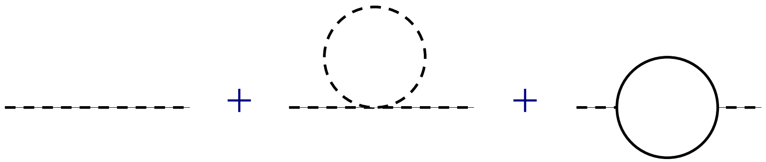


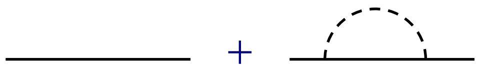
radiative correction of masses

$$M^2 = M_0^2 + \#\Lambda^2 + \# m_0^2 \log\frac{\Lambda}{m_0} + \cdots$$

$$m = m_0 + \# m_0 \log\frac{\Lambda}{m_0} + \cdots$$

Weisskopf '34



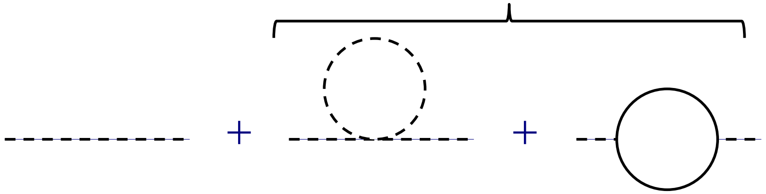


extremely-fine tuning needed to account for M_H=125GeV << Λ~10^{17}GeV

Hierarchy problem of SM

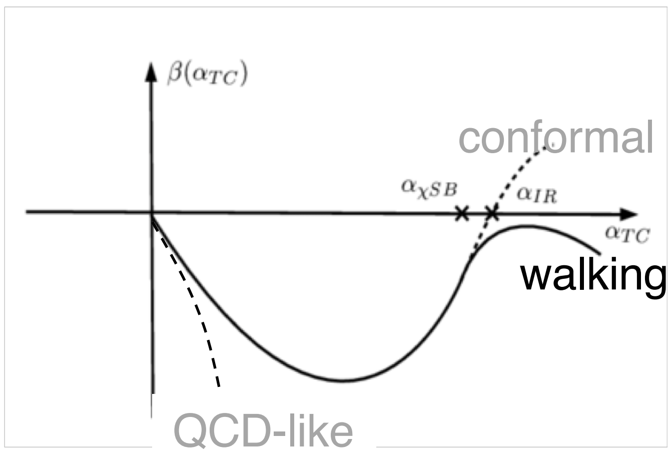
Solution 1: SUSY $\left(H\ ,\ \psi_H\right)$ degenerate m_0 ⇨ degenerate m

$$m^2 = m_0^2 + \underset{\substack{\uparrow \\ \text{O}(\Lambda^2)\text{ cancelled}}}{0} + \# m_0^2 \log\frac{\Lambda}{m_0} + \cdots \qquad\qquad m = m_0 + \# m_0 \log\frac{\Lambda}{m_0} + \cdots$$



Solution 2: Higgs is ~~fundamental~~ ⇨ composite of 2 fermions in (new) gauge theory

$H \sim \bar{t}\, t$ (top condensation) or $\bar{Q}\, Q$ (technicolor)



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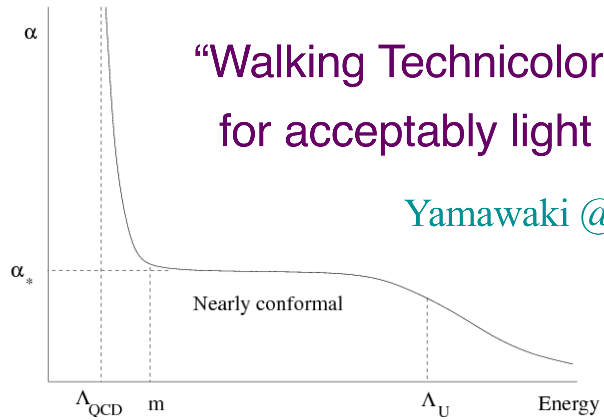


"Walking Technicolor"

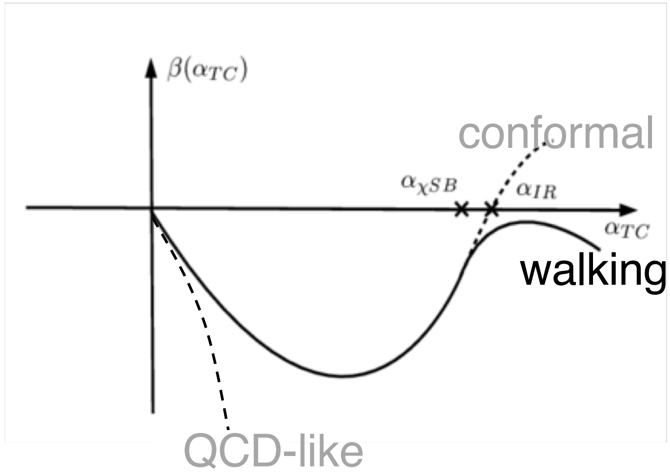
for acceptably light m_H

Yamawaki @Nagoya

Hierarchy problem of SM

Solution 2: Higgs is ~~fundamental~~ ⇨ composite of 2 fermions in new gauge theory

QCD × $10^{\geq 3}$ scale-up



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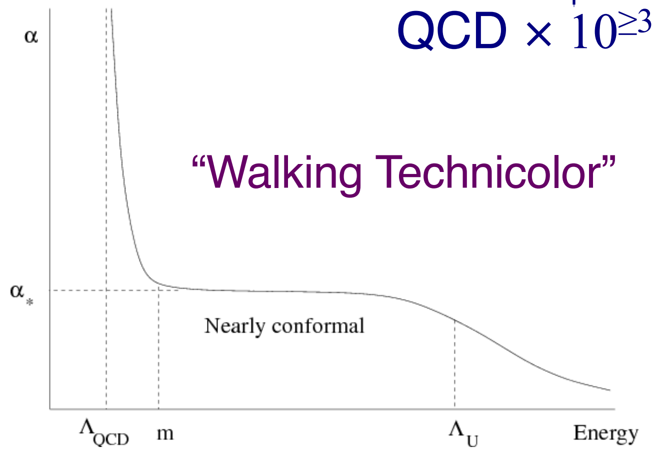

"Walking Technicolor" Yamawaki

$$\beta(\alpha) = -\underbrace{\left(\frac{11}{3}N - \frac{2}{3}n_F - \frac{4}{3}N\,n_{Ad}\right)}_{\text{small but }>0}\alpha^2 - \underbrace{\left(\frac{34}{3}N^2 - \left(\frac{13}{3}N - \frac{1}{N}\right)n_F - \frac{32}{3}N^2 n_{Ad}\right)}_{<0}\alpha^3 - \cdots$$

- $N = 3\ ,\ n_F = 12$ Nagoya KMI group; Kuti; Hasenfratz
- $N = 2\ ,\ n_{Ad} = 2$ "Minimal WTC" Catterall-Sannino
- $N = 2\ ,\ n_F = 8$ NCTU group, Helsinki group...

Chiral symm. broken like QCD

Conformal (unsuited for WTC) ?

…tested for $\mathrm{SU}(2)$ GTs on lattice

Chiral Symm Breaking in QCD

$$\langle \cdots \rangle_{\rm QCD} = \frac{1}{Z(m)} \int DA_\mu \prod_{f=1}^{n_f} D\bar\psi_f D\psi_f \ \exp\left\{-S_{\rm YM}[A_\mu] + \int \bar\psi_f (\not{D} + m_f)\psi_f\right\} \cdots \ , \quad \not{D} = \left[\begin{array}{c|c} 0 & \sigma_\mu D_\mu \\ \hline \bar\sigma_\mu D_\mu & 0 \end{array}\right]$$

$$= \frac{1}{Z(m)} \int DA_\mu \ e^{-S_{YM}[A]} \prod_{f=1}^{n_f} \det\left(\not{D} + m_f\right) \ \cdots \qquad [\dim(\mathrm{Ker}\,\not{D}) = \nu \quad \text{sector}]$$

Dirac EVs $\simeq$ imag. mass

flavor symm of massless lim. $G = \begin{cases} \mathrm{SU}(2n_f) & \text{ps-real rep} \\ \mathrm{SU}(n_f)\times\mathrm{SU}(n_f) & \text{complex rep} \\ \mathrm{SU}(2n_f) & \text{real rep} \end{cases}$ $\xrightarrow[\text{is dynamically formed}]{\text{broken if } \Sigma = \langle \bar\psi_f \psi_f \rangle_{\rm QCD}}$ $H = \begin{cases} \mathrm{Sp}(2n_f) \\ \mathrm{SU}(n_f) \\ \mathrm{SO}(2n_f) \end{cases}$

A_μ, ψ_f confined under $T_{\rm C}$ ⇨ NG boson (pion) $U_{ff'}$ dominance in the low-energy regime

$$Z_{\rm eff}(m) = \int_{G/H\times \mathrm{U}(1)} DU \ \exp\left\{-\int F_\pi^2 \ \mathrm{tr}\ \partial_\mu U^+ \partial_\mu U - \Sigma \ \mathrm{Re\ tr}\left(\mathrm{diag}(m_f)U\right) - \nu \log\det U\right\}$$

effective chiral L constrained by chSB, F_π and Σ : undetermined

on finite lattice V=L^4 & at small masses

s.t. $F_\pi^2 L^{-2} >> \Sigma m$

$$Z_{\text{0-mode}}(m) = \int_{G/H\times \mathrm{U}(1)} dU \ \left(\det U\right)^\nu \exp\left\{V\Sigma \ \mathrm{Re\ tr\ diag}(m_f)U\right\}$$

Chiral Random Matrices

[Shuryak-Verbaarschot '93]

$$\langle \cdots \rangle_{\mathrm{chRM}} = \frac{1}{Z(m)} \int dH \; \mathrm{e}^{-\mathrm{tr}\, H^2} \prod_{f=1}^{n_f} \det\left(H + i m_f\right) \cdots \quad , \quad H = \begin{array}{cc} & \overbrace{}^{N+\nu} \\ \left[\begin{array}{c|c} 0 & M \\ \hline M^+ & 0 \end{array} \right] & \begin{array}{l} \} N \\ \\ \end{array} \end{array}$$

$$M_{ab} \in \begin{cases} \mathbf{R} : \beta = 1 \\ \mathbf{C} : \beta = 2 \\ \mathbf{H} : \beta = 4 \end{cases}$$

HS transf.

$N \to \infty \;\; , \;\; m_f \to 0$

$\mu_f = N m_f \;\; \text{fixed}$

captures all
Global & Discrete
symmetries of QCD

0D reduction of chL

$$\int_{U(N_F)} dU \; \left(\det U\right)^{\nu} \exp\left\{\mathrm{Re}\;\mathrm{tr}\;\mathrm{diag}(\mu_f) U\right\} = \mathrm{cst.} \frac{\det\left[\mu_i^{j-1} I_{\nu+j-i}(\mu_i)\right]_{i,j=1}^{N_F}}{\Delta(\mu_f^2)}$$

for β=2 [Brower-Rossi-Tan '81]

similar forms with $\mathrm{Pf} \sim \mathrm{qdet}$ for β=1,4 [Smilga-Verbaarschot '95, Nagao-SMN '00]

analytically solvable, symmetry-based model of QCD in **chSB phase**

Chiral Random Matrices

[Shuryak-Verbaarschot '93]

$$\left\langle \cdots \right\rangle_{\rm chRM} = \frac{1}{Z(m)} \int dH \; e^{-\mathrm{tr}\, H^2} \prod_{f=1}^{n_f} \det\left(H + i m_f\right) \cdots \quad , \quad H = \begin{array}{cc} & \overbrace{}^{N+\nu} \\ \left[\begin{array}{c|c} 0 & M \\ \hline M^+ & 0 \end{array}\right] & \}\, N \end{array}$$

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captures all
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symmetries of QCD

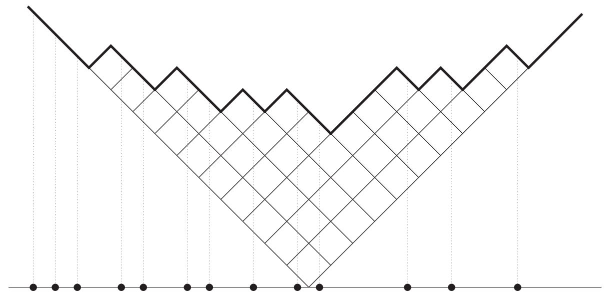
$$\int_{U(N_F)} dU \; \left(\det U\right)^{\nu} \exp\left\{\mathrm{Re}\,\mathrm{tr}\,\mathrm{diag}(\mu_f) U\right\} = \mathrm{cst.} \frac{\det\left[\mu_i^{j-1} I_{\nu+j-i}(\mu_i)\right]_{i,j=1}^{N_F}}{\Delta(\mu_f^2)}$$

effective chL of QCD: $V \Sigma \, \mathrm{Re}\,\mathrm{tr}\,\mathrm{diag}(m_f)\, U$

if QCD is in **chSB phase**,

- Dirac EVDs on various V collapse onto chRM result
- can determine Σ by fitting

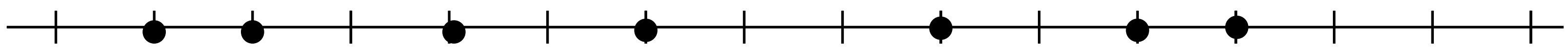
Det. (fermionic) point process



- Plancherel measure {YT}
- directed percolation
- continuous ⇨ β=2 RM

$$\mathrm{Prob}_N(n_1,\ldots,n_N) = \frac{1}{N!}\det\left[K(n_i,n_j)\right]_{i,j=1}^{N} \qquad n_i \in \mathbf{Z}$$

$$\mathbf{K} = \left[K(n,m)\right]_{n,m\in\mathbf{Z}} \ : \text{projective } \mathbf{K}\cdot\mathbf{K} = \mathbf{K}\ ,\ \mathrm{tr}\,\mathbf{K} = N\,,\ \ \mathbf{K}^{\dagger}=\mathbf{K}$$



then,

$$R_{N-1}(n_1,\ldots,n_{N-1}) = N\sum_{m\in\mathbf{Z}}\frac{1}{N!}\det\begin{bmatrix}\left[K(n_i,n_j)\right]_{i,j=1}^{N-1} & \left[K(m,n_j)\right]_{j=1}^{N-1}\\ \left[K(n_i,m)\right]_{i=1}^{N-1} & K(m,m)\end{bmatrix} = \frac{N-(N-1)}{(N-1)!}\det\left[K(n_i,n_j)\right]_{i,j=1}^{N-1}$$

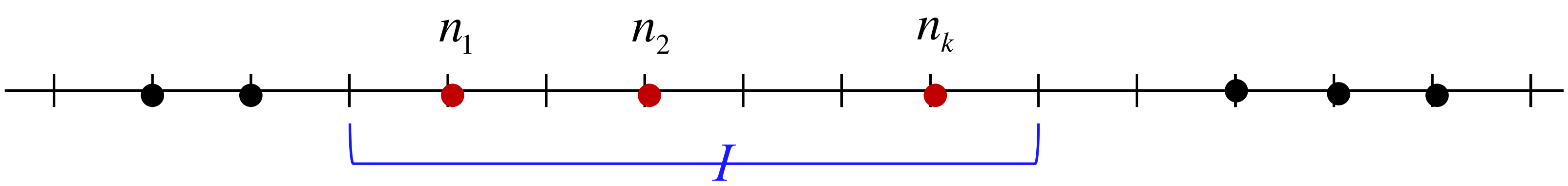
$\Downarrow$ repeat

$$R_k(n_1,\ldots,n_k) = \det\left[K(n_i,n_j)\right]_{i,j=1}^{k} \qquad = \text{repeat} \Rightarrow \qquad R_1(n) = K(n,n)$$

Janossy density

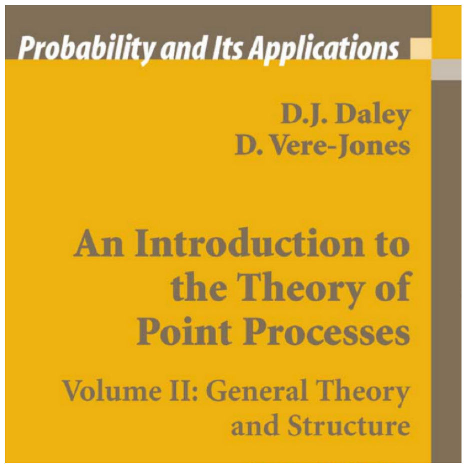
for Det point process $R_k(n_1,\dots,n_k) = \det\left[K(n_i,n_j)\right]_{i,j=1}^{k}$

$J_{k,I}(n_1,\dots,n_k) =$ Prob of



no pts in I except for k designated pts

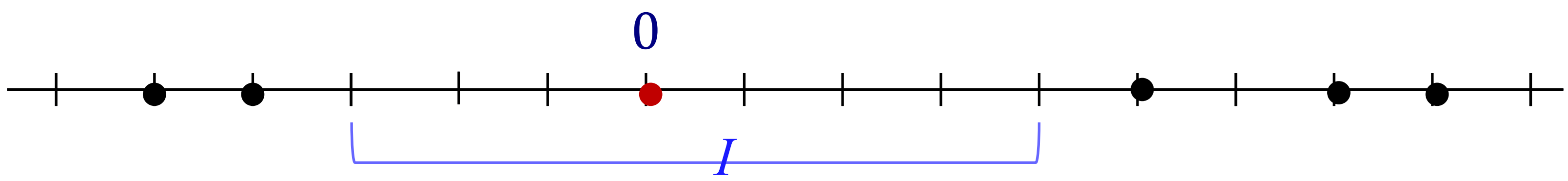
[Textbook '88]



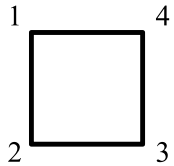
$$\mathbf{K}_I := \left[K(n,m)\right]_{n,m\in I}$$

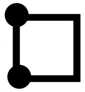
$$J_{k,I}(n_1,\dots,n_k) = \det\overbrace{\left(1-\mathbf{K}_I\right)}^{\text{all pts in } I}\cdot\det\left[\overbrace{\left\langle n_i\middle|\mathbf{K}_I\left(1-\mathbf{K}_I\right)^{-1}\middle|n_j\right\rangle}^{\text{designated pts } \bullet}\right]_{i,j=1}^{k}$$

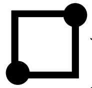
Proof



$$
\begin{aligned}
& J_{1,I}(0) \\
= \ & R_1(0) - \sum_{n\in I} R_2(0,n) + \frac{1}{2!}\sum_{n,n'\in I} R_3(0,n,n') - \cdots \\
= \ & K(0,0) - \sum_{n\in I}\begin{vmatrix} K(0,0) & K(0,n) \\ K(n,0) & K(n,n) \end{vmatrix} + \frac{1}{2!}\sum_{n,n'\in I}\begin{vmatrix} K(0,0) & K(0,n) & K(0,n') \\ K(n,0) & K(n,n) & K(n,n') \\ K(n',0) & K(n',n) & K(n',n') \end{vmatrix} \cdots \\
= \ & \langle 0|\mathbf{K}_I|0\rangle - \left\{\langle 0|\mathbf{K}_I|0\rangle \ \mathrm{tr}\mathbf{K}_I - \left\langle 0|\mathbf{K}_I^2|0\right\rangle\right\} \\
& + \frac{1}{2!}\left\{\langle 0|\mathbf{K}_I|0\rangle \left(\mathrm{tr}\mathbf{K}_I\right)^2 - \langle 0|\mathbf{K}_I|0\rangle \ \mathrm{tr}\mathbf{K}_I^2 - 2\left\langle 0|\mathbf{K}_I^2|0\right\rangle \mathrm{tr}\mathbf{K}_I + 2\left\langle 0|\mathbf{K}_I^3|0\right\rangle\right\} - \cdots \\
= \ & \langle 0|\mathbf{K}_I|0\rangle \left\{1 - \mathrm{tr}\mathbf{K}_I + \frac{1}{2!}\left(\mathrm{tr}\mathbf{K}_I\right)^2 - \cdots\right\}\left\{1 - \frac{1}{2}\mathrm{tr}\mathbf{K}_I^2 + \cdots\right\}\cdots \\
+ \ & \left\langle 0|\mathbf{K}_I^2|0\right\rangle \left\{1 - \mathrm{tr}\mathbf{K}_I + \cdots\right\}\cdots \\
+ \ & \left\langle 0|\mathbf{K}_I^3|0\right\rangle \left\{1 - \cdots\right\}\cdots + \cdots \\
= \ & \left\langle 0|\mathbf{K}_I + \mathbf{K}_I^2 + \mathbf{K}_I^3 + \cdots|0\right\rangle \det(\mathbf{1} - \mathbf{K}_I) = \left\langle 0|\mathbf{K}_I(\mathbf{1}-\mathbf{K}_I)^{-1}|0\right\rangle \det(\mathbf{1}-\mathbf{K}_I)
\end{aligned}
$$

Example: 2 Fermions on 

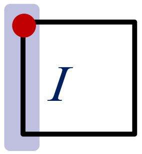
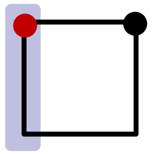
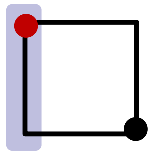
Prob() = 1/8

Prob() = 1/4

$$\Longleftrightarrow \quad \mathrm{Prob}(n,m) = \frac{1}{2!}\begin{vmatrix} K(n,n) & K(n,m) \\ K(m,n) & K(m,m) \end{vmatrix}$$

$$\mathbf{K} = \frac{1}{4}\begin{pmatrix} 2 & 1-i & 0 & 1+i \\ 1+i & 2 & 1-i & 0 \\ 0 & 1+i & 2 & 1-i \\ 1-i & 0 & 1+i & 2 \end{pmatrix} = \mathbf{K}\cdot\mathbf{K}$$

$\mathrm{tr}\,\mathbf{K} = 2$

$J_{1,I}(1) = \mathrm{Prob}($  $) =$  $+$  $= \frac{1}{8} + \frac{1}{4}$

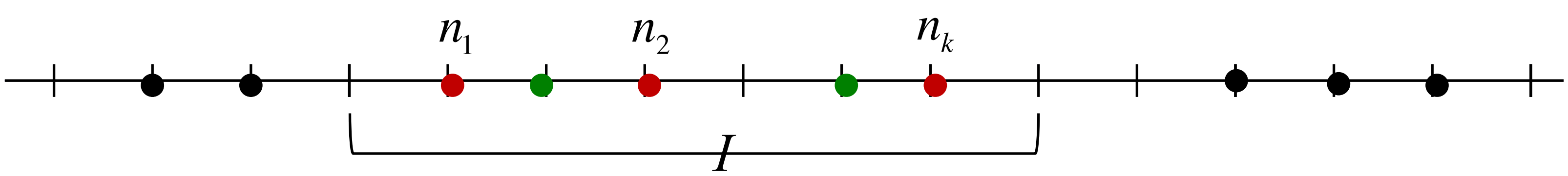
$$= \det(\mathbf{1}-\mathbf{K}_I)\cdot\langle 1|\mathbf{K}_I(\mathbf{1}-\mathbf{K}_I)^{-1}|1\rangle$$

$$= \left|\mathbf{1} - \tfrac{1}{4}\begin{pmatrix} 2 & 1-i \\ 1+i & 2 \end{pmatrix}\right| \cdot \begin{pmatrix} 3 & 2-2i \\ 2+2i & 3 \end{pmatrix}_{1,1} = \frac{1}{8}\cdot 3$$

Janossy density

Det. point process: $R_k(n_1,\ldots,n_k) = \det\left[K(n_i,n_j)\right]_{i,j=1}^{k}$

$J_{p,k,I}(n_1,\ldots,n_k)$ = Prob of



exactly p pts ● in I except for k designated pts ●

$$\mathbf{K}_I := \left[K(n,m)\right]_{n,m\in I}$$

$$J_{p,k,I}(n_1,\ldots,n_k) = \frac{1}{p!}\left(-\partial_\xi\right)^p \det\left(1-\xi\mathbf{K}_I\right)\cdot\det\left[\left\langle n_i\middle|\mathbf{K}_I\left(1-\xi\mathbf{K}_I\right)^{-1}\middle|n_j\right\rangle\right]_{i,j=1}^{k}\Bigg|_{\xi=1}$$

2 Janossy density in DPP

- In case $(\mathbf{1}-\mathbf{K}_I)$ may be singular : use $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |D|\,|A-CD^{-1}B|$

$$J_{k,I}(n_1,\dots,n_k) = \det(\mathbf{1}-\mathbf{K}_I)\cdot\det\left[\langle n_i|\mathbf{K}_I(\mathbf{1}-\mathbf{K}_I)^{-1}|n_j\rangle\right]_{i,j=1}^{k}$$

$$= (-)^k \det\begin{vmatrix} \overbrace{-\left[\langle n_i|\mathbf{K}_I|n_j\rangle\right]_{i,j=1}^{k}}^{\text{designated pts } \bullet} & \overbrace{-\left[\langle m|\mathbf{K}_I|n_j\rangle\right]_{j=1,\dots,k}^{m\in I}}^{\text{all pts in } I} \\ -\left[\langle n_i|\mathbf{K}_I|m\rangle\right]_{m\in I}^{j=1,\dots,k} & \mathbf{1}-\mathbf{K}_I \end{vmatrix}\begin{array}{l} \}\ \bullet \\ \}\ I \end{array}$$

unifies 2 well-known formulas in DPP / RM

$$R_k(n_1,\dots,n_k) = \det[K(n_i,n_j)]_{i,j=1}^{k} \qquad : \ k\text{-point Corr. Function}$$

$$E_{0,I} = \det(\mathbf{1}-\mathbf{K}_I) \qquad : \text{Gap Probability}$$

- Generalizable to qdet : Pfaffian PP / β=1&4 RM

2 Janossy density in DPP

- In case $(\mathbf{1}-\mathbf{K}_I)$ may be singular : use $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |D|\,|A - CD^{-1}B|$

$$J_{k,I}(n_1,\dots,n_k) = \det(\mathbf{1}-\mathbf{K}_I)\cdot\det\left[\langle n_i|\mathbf{K}_I(\mathbf{1}-\mathbf{K}_I)^{-1}|n_j\rangle\right]_{i,j=1}^{k}$$

$$= (-)^k \det\begin{vmatrix} \overbrace{-\left[\langle n_i|\mathbf{K}_I|n_j\rangle\right]_{i,j=1}^{k}}^{\text{designated pts}\ \bullet} & \overbrace{-\left[\langle m|\mathbf{K}_I|n_j\rangle\right]_{j=1,\dots,k}^{m\in I}}^{\text{all pts in } I} \\ -\left[\langle n_i|\mathbf{K}_I|m\rangle\right]_{m\in I}^{j=1,\dots,k} & \mathbf{1}-\mathbf{K}_I \end{vmatrix}\begin{matrix} \}\ \bullet \\ \}\ I \end{matrix}$$

- Continuous distribution : Fredholm Det approx. by $I \sim \{y_1,\dots,y_m\}$

$$K(m,m') \to \sqrt{\Delta y_a}K(y_a,y_b)\sqrt{\Delta y_b}$$

$$J_{k,I}(x_1,\dots,x_k) = \lim_{\Delta y_a\to 0}$$

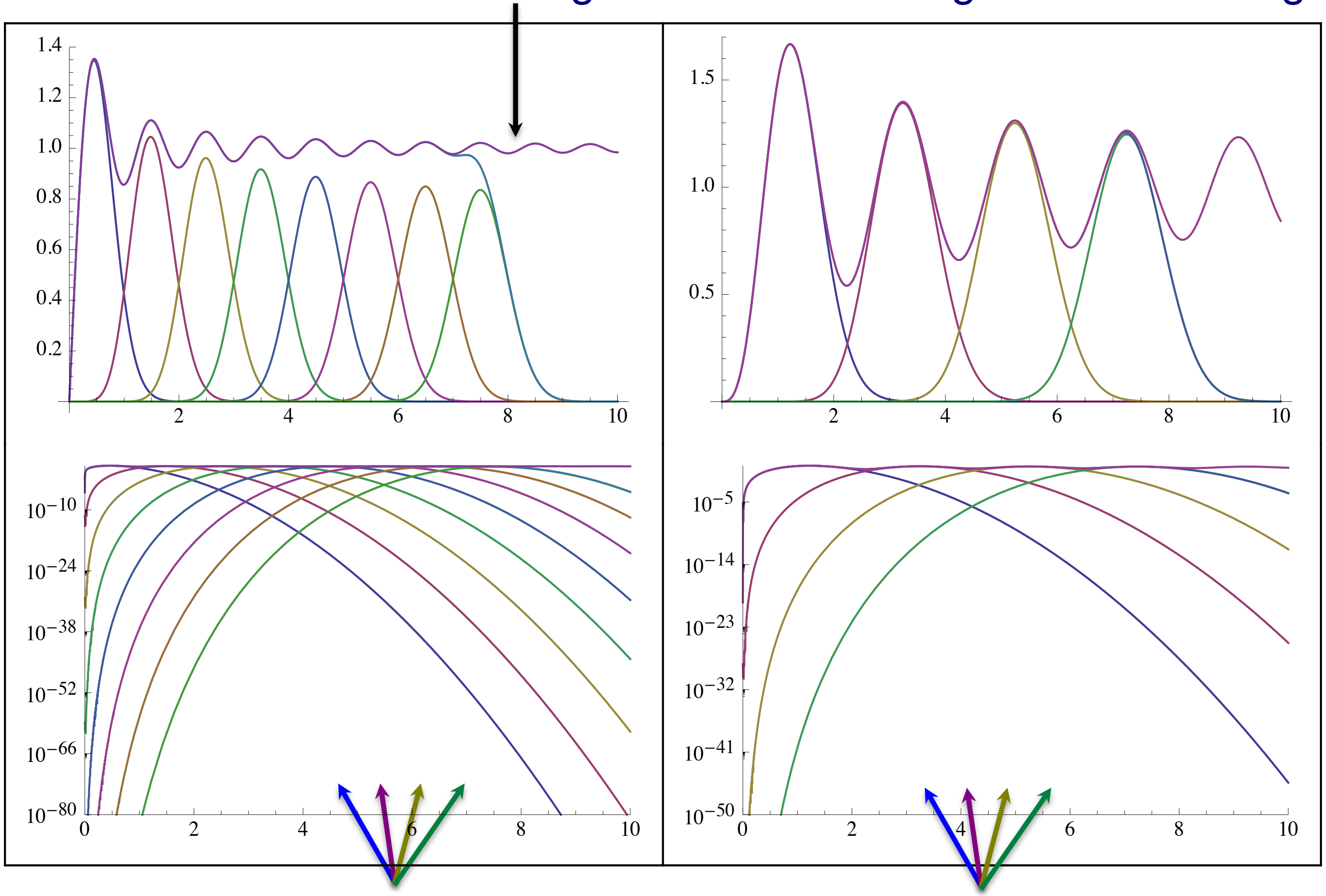
$$(-)^k \det\begin{vmatrix} -\left[K(x_i,x_j)\right]_{i,j=1}^{k} & -\left[\sqrt{\Delta y_a}K(y_a,x_i)\right]_{y_a\in I}^{i=1,\dots,k} \\ -\left[K(x_i,y_b)\sqrt{\Delta y_b}\right]_{y_b\in I}^{j=1,\dots,k} & \mathbf{1}-\left[\sqrt{\Delta y_a}K(y_a,y_b)\sqrt{\Delta y_b}\right]_{y_a,y_b\in I} \end{vmatrix}$$

new? not explicit in [Borodin-Soshnikov ’03][Forrester-Witte ’07][Forrester-Witte-Bornemann ’12]

Individual EV distributions

1st ～8th EV distributions of chGUE , chGSE

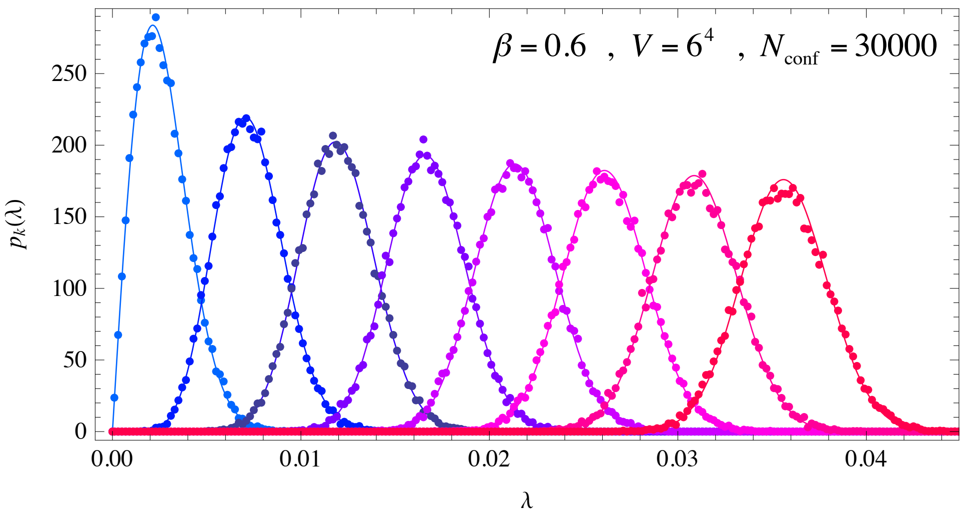
flattening oscillation ⇒ larger error for fitting

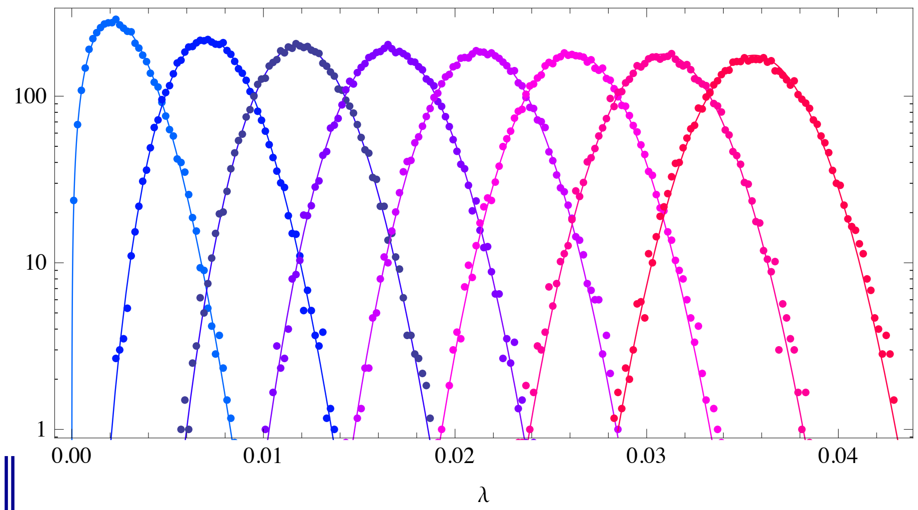


quasi-Gaussian ⇒ precise fitting possible

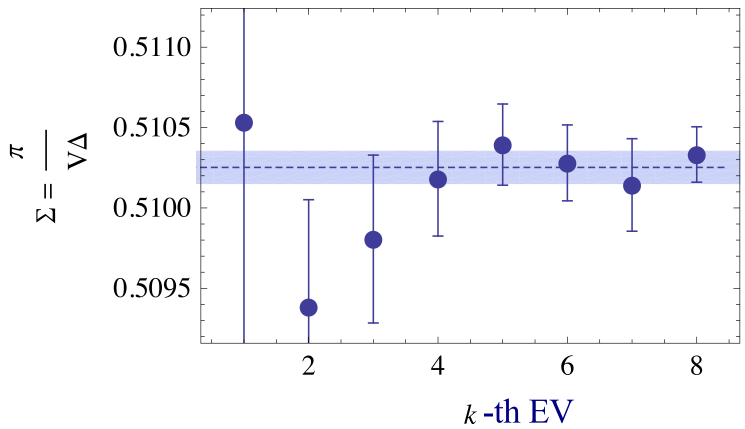
Chiral condensate from Individual EV distributions

exercise 1 : quenched U(1) Dirac spectrum vs chGUE





chiral condensate

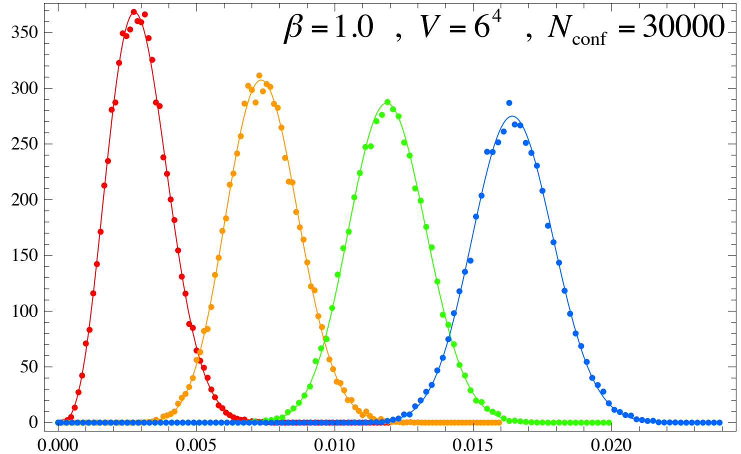


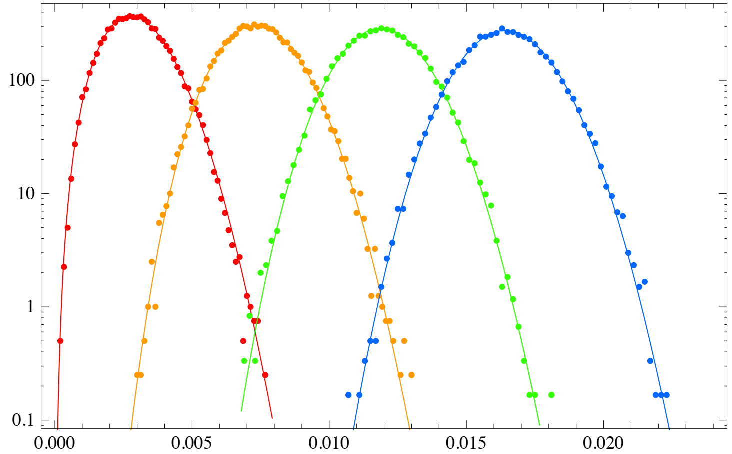
[SMN '16]

$$\Sigma a^3 = 0.51025(10)$$

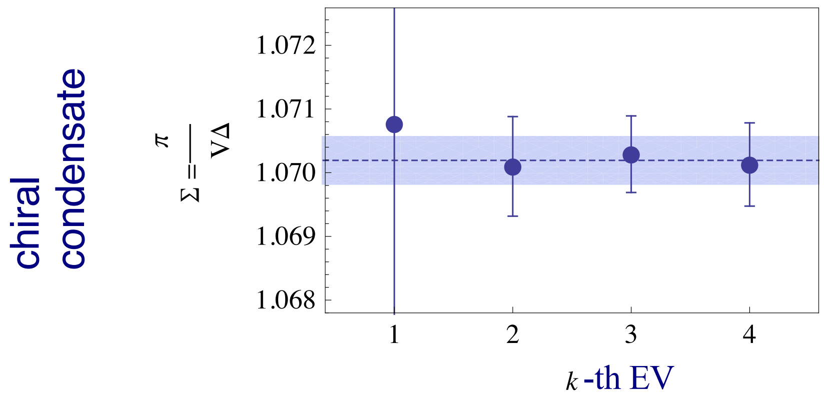
Chiral condensate from Individual EV distributions

exercise 2 : quenched SU(2) Dirac spectrum vs chGSE





⇓



[SMN '16]

$$\Sigma a^3 = 1.07019(38)$$

$\boxed{\Sigma \text{ within } O(10^{-4}) \text{ error!}}$

Technical problems

[Damgaard-SMN '01]

"Shifting" method for chRM

∂(rect.matrix)/∂(sing.values)

$\lambda_i \geq 0 \in \mathrm{Spec}(H^2)$

$$dH\ \mathrm{e}^{-\mathrm{tr}\,H^2} \prod_f \det\left(H + i m_f\right) \propto \prod_{i=1}^{N} \left(d\lambda_i\ \lambda_i^{\beta(\nu+1)/2-1}\ \mathrm{e}^{-\lambda_i} \prod_f \left(\lambda_i + m_f^2\right)\right) \prod_{i>j}^{N} \left|\lambda_i - \lambda_j\right|^{\beta}$$ … JPD of EVs

$$P_k(\lambda_1,\ldots,\lambda_k) = \int_{\lambda_k}^{\infty} \cdots \int_{\lambda_k}^{\infty} d\lambda_{k+1} \cdots d\lambda_N \left(JPD_{N_F}(\lambda_1,\ldots,\lambda_N;\{m\};\nu)\right)$$ … JPD of first k EVs

$$\tilde{\lambda}_i \equiv \lambda_i - \lambda_k \quad \Downarrow$$

$$= C(\{m\}) \int_0^{\infty} \cdots \int_0^{\infty} d\tilde{\lambda}_{k+1} \cdots d\tilde{\lambda}_N \left(JPD_{\tilde{N}_F}(\tilde{\lambda}_1,\ldots,\tilde{\lambda}_N;\{\tilde{m}\};\tilde{\nu})\right) \xrightarrow{N\to\infty} \text{ratio of Bessel det's}$$

$$\Downarrow$$

$$p_k(\lambda_k) = \int_0^{\lambda_k} \cdots \int_0^{\lambda_k} d\lambda_1 \cdots d\lambda_{k-1} P_k(\lambda_1,\ldots,\lambda_k) \xrightarrow{N\to\infty} \boxed{(k-1)\text{-fold integral of ratio of Bessel det's}}$$

for this trick to work, the exponent

$$\beta\frac{\nu+1}{2} - 1 \in \begin{cases} \mathbf{N} & (\beta = 1,2) \\ 2\mathbf{N} & (\beta = 4) \end{cases} \Rightarrow \begin{array}{l} \times\ \mathrm{chGOE},\ \nu = 0,2,4\ldots \\ \times\ \mathrm{chGSE},\ N_F = 0,2,4\ldots \end{array}$$

practically unfeasible for $k \geq 5$

Nystrom-type approx to Fredholm Det

Gauss-Legendre Quadrature : $\{x_1,\ldots,x_M\}\in I,\quad \{\Delta x_1,\ldots,\Delta x_M\}>0$

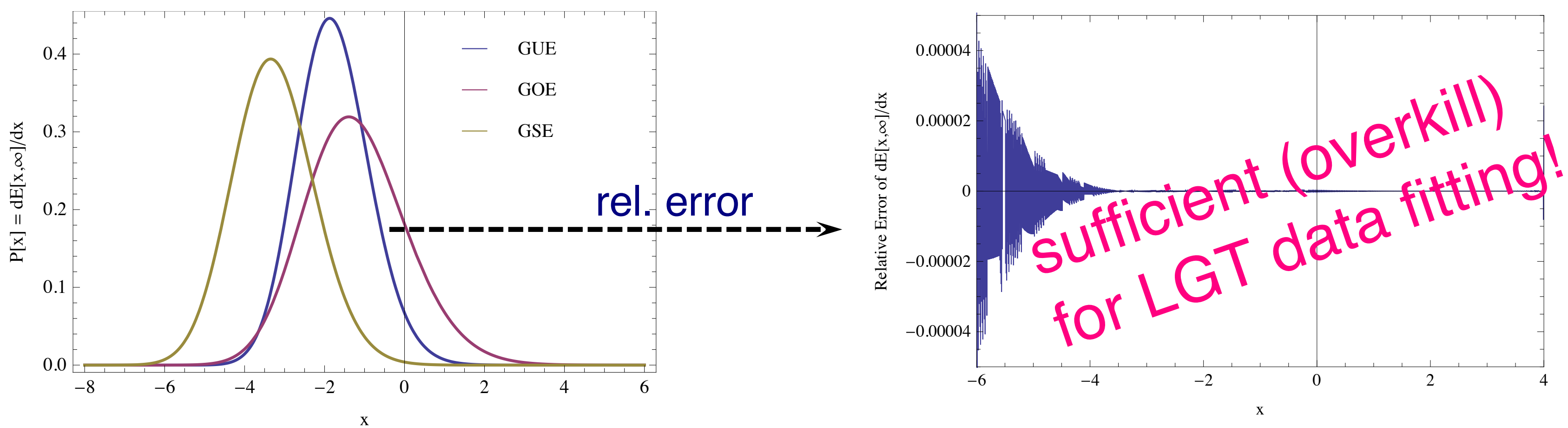
$$\int_I f(x)\,dx \;\cong\; \sum_{i=1}^{M} f(x_i)\Delta x_i \quad \text{, exact for } f(x)=x^M+\text{lower}$$

$$\boxed{\operatorname{Det}(1-K_I) \;\cong\; \det\left[\delta_{ij}-K(x_i,x_j)\sqrt{\Delta x_i \Delta x_j}\right]_{i,j=1}^{M} \quad +\text{relative error } O(\mathrm{e}^{-\text{const}.M})}$$

[Bornemann '10]

ex. Largest EV distribution

Nystrom approx (M=30) for K_{Airy} vs Tracy-Widom's analytic formula



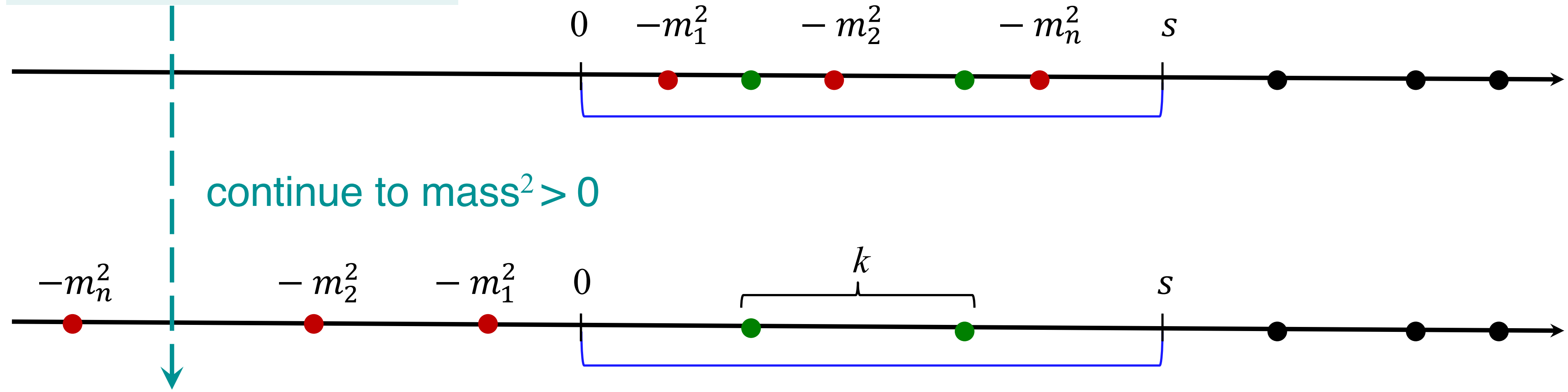
$n_F = \beta\, n$ fermions as Janossy density

$$\int dH\, \mathrm{e}^{-\mathrm{tr}\, H^2} \prod_{f=1}^{n} (H + i m_f)^{\beta} = \int_0^{\infty} \prod_{i=1}^{N} \left(d\lambda_i\, \lambda_i^{\frac{\beta(\nu+1)}{2}-1} \mathrm{e}^{-\lambda_i} \prod_{f=1}^{n} \left| \lambda_i + m_f^2 \right|^{\beta} \right) \prod_{i>j}^{N} \left| \lambda_i - \lambda_j \right|^{\beta}$$

$$\propto \int_0^{\infty} \prod_{i=1}^{N+n} \left(d\lambda_i\, \lambda_i^{\frac{\beta(\nu+1)}{2}-1} \mathrm{e}^{-\lambda_i} \right) \prod_{i>j}^{N+n} \left| \lambda_i - \lambda_j \right|^{\beta} \cdot \prod_{k=N+1}^{N+n} \delta\left(\lambda_k - (-m_f^2)\right)$$

designated pts ● = −mass² > 0

$J_{k,n,[0,s]}(-m_1^2, \dots, -m_n^2)$

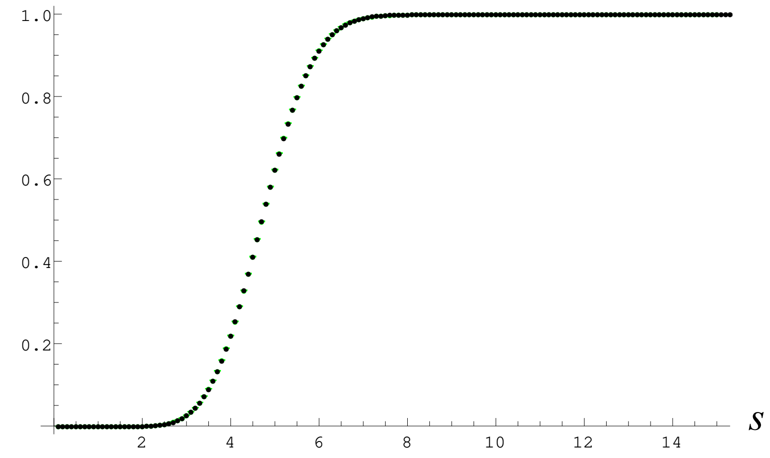


$E_k(s; m_1^2, \dots, m_n^2)$

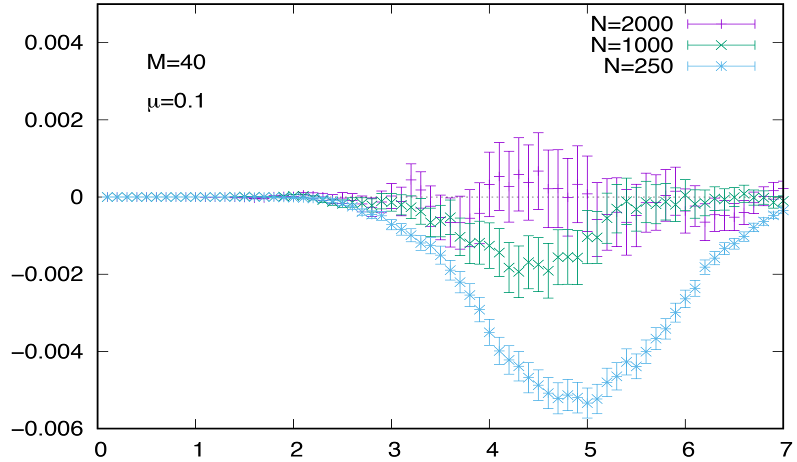
$\Longrightarrow$ evaluate Fredholm Det by quadrature approx.

$\beta = 4$ (chGSE), $n_F = 4$, $k = 0$, $\mu = 0.1$ vs chGSE (N= 250~2000) by HMC

$1\text{-}E_0(s\,;0.1)$



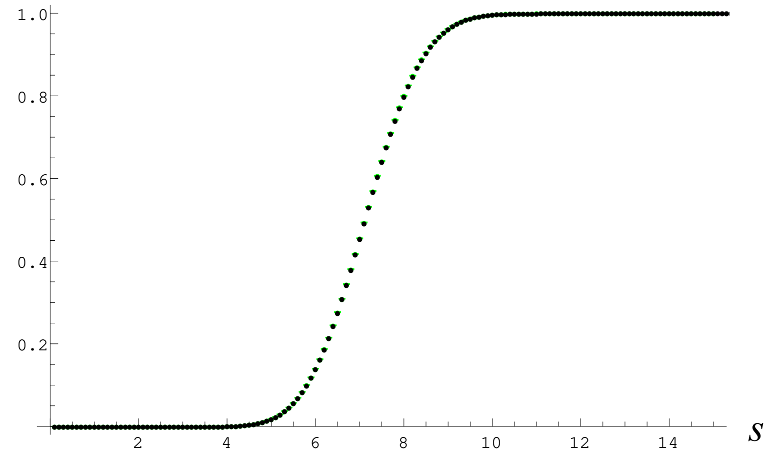
deviation



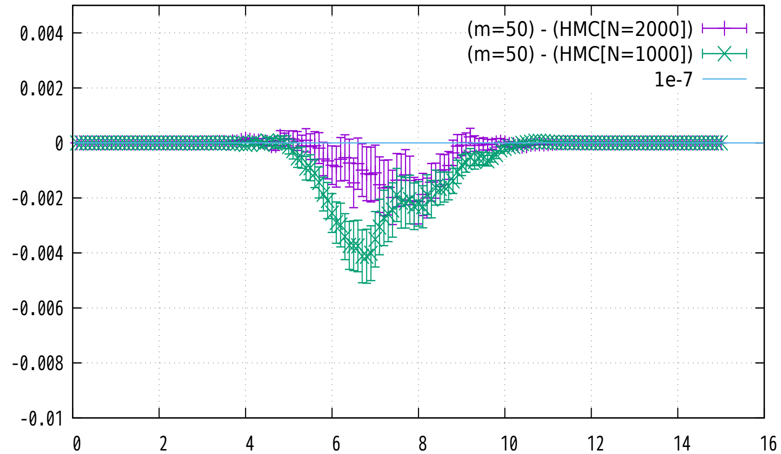
Quadrature Approx of Det : systematic error too small for bare eyes!

$\beta = 4$ (chGSE), $n_F = 8$, $k = 0$, $\mu = 0.1$ vs chGSE (N= 1000~2000) by HMC

$1\text{-}E_0(s\,;0.1)$



deviation

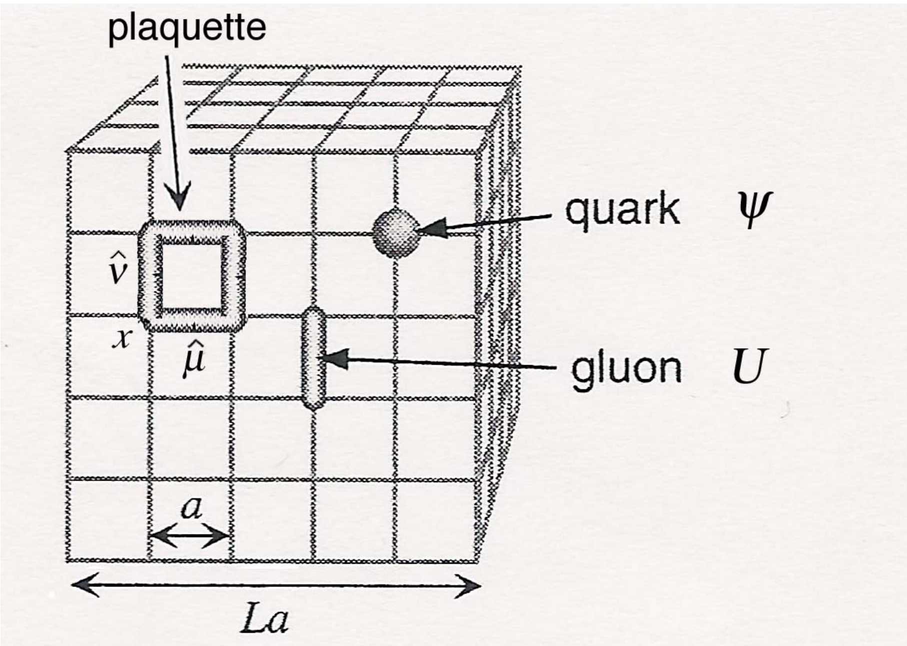


Judgement of chSB for WTC candidate

Lattice QCD with staggered quarks

$$Z = \int \prod_{x\in X,\, y\in O_x} dU_{xy} \int \prod_{x} d\bar{\psi}_x d\psi_x \prod_{\mathrm{plaq}} e^{\,2N/g2\ \mathrm{tr}\ UUUU}$$

$$\exp\left\{ \sum_{\langle x,y \rangle} \frac{\eta_{xy}}{2} \left(\bar{\psi}_x^{i,f} U_{xy}^{ij} \psi_y^{j,f} - \bar{\psi}_y^{i,f} U_{xy}^{+ij} \psi_x^{j,f} \right) - \sum_x \bar{\psi}_x^{i,f} m^{fg} \psi_x^{i,g} \right\}$$



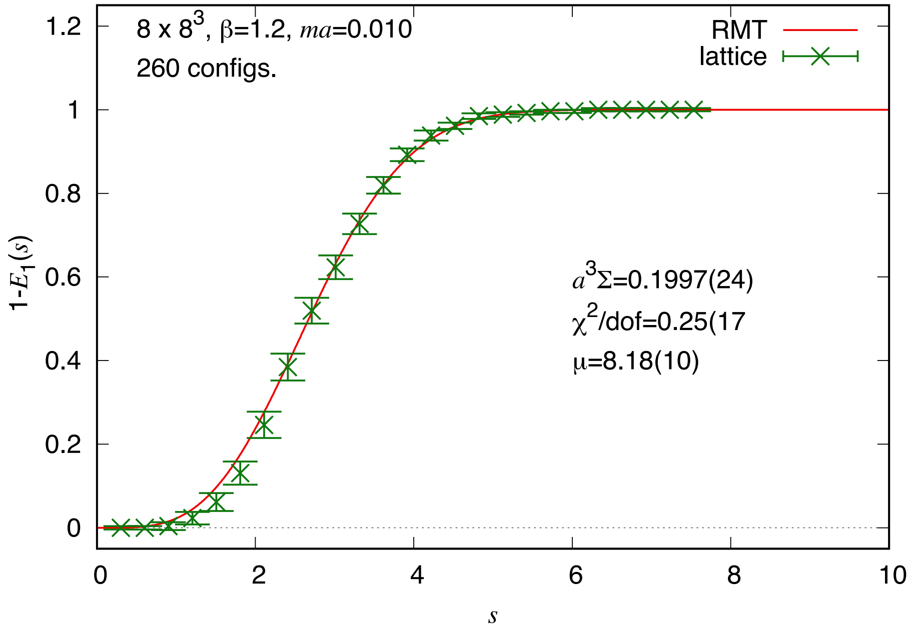
- bipartite → L / R chirality & spinor
- quark doublers : $2^{d/2} = 4$ ‘tastes’
- taste-degeneracy is broken by $O(a^2)$ lattice artifact

Judgement of chSB for WTC candidate

flavor taste

$\beta = 4$ (chGSE), $n_F = 4$, $k = 0$ vs 2C QCD, $n_F = 2 \times 4 = 2+(2+2+2)$

$\mu = 8.18$ $ma = 0.010$



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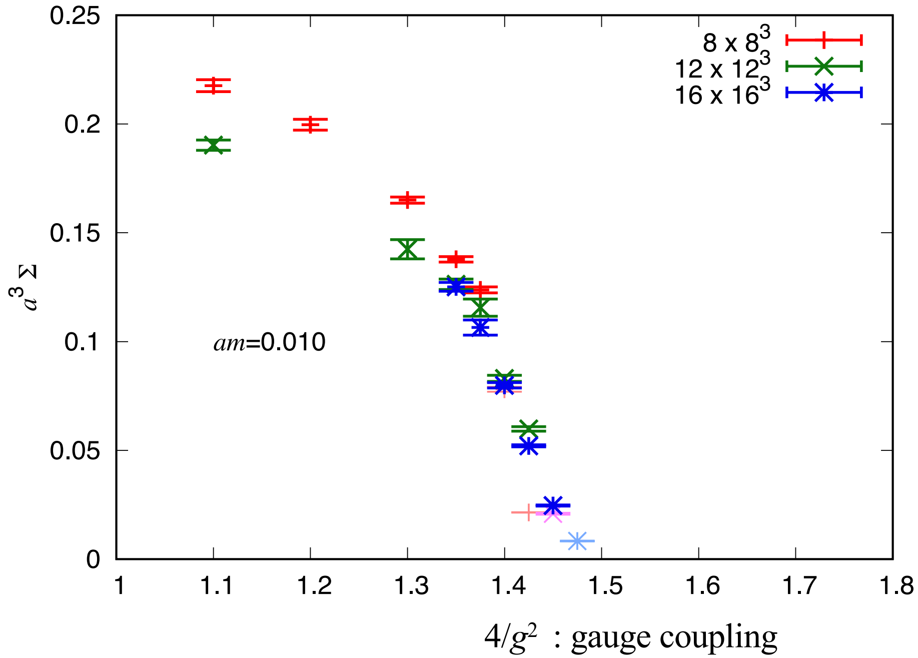
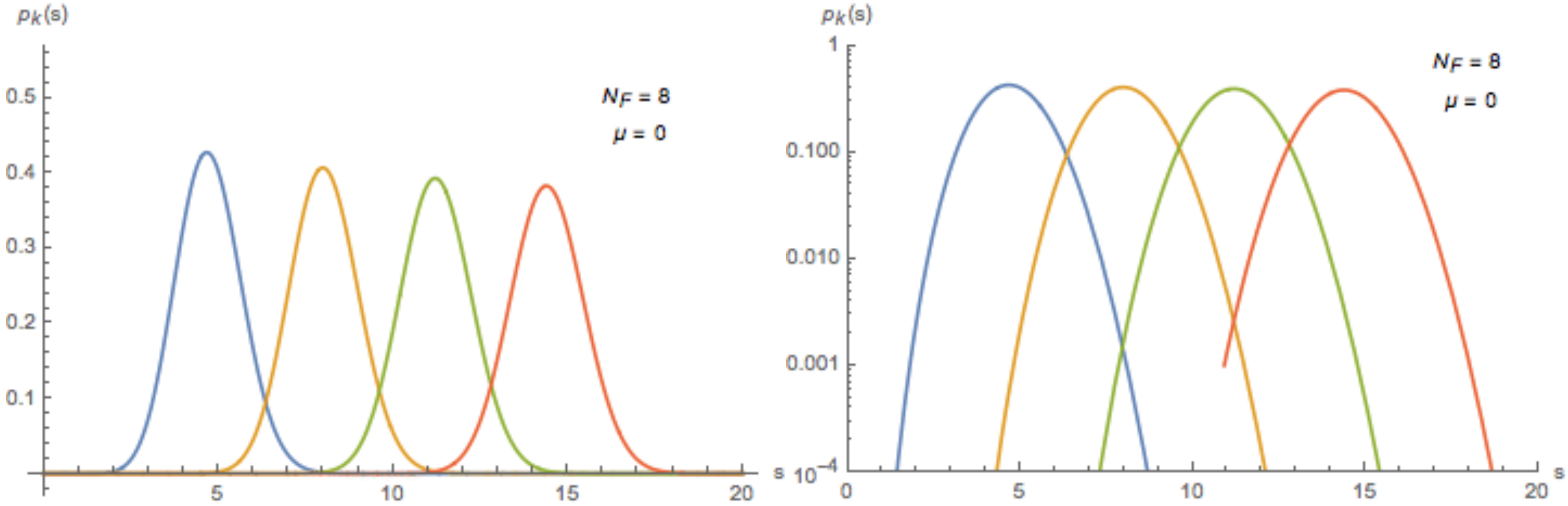


N=2, $n_F = 2+(2+2+2)$, $\beta \leq 1.4\ldots$: Chiral Symm Broken

Judgement of chSB for WTC candidate

$\beta = 4$ (chGSE), $n_F = 8$, $k = 0,1,2,3$, $\mu = 0,1,2,\ldots,\infty$



to be fitted with

flavor taste

$N = 2,\ n_F = 8 \times 1$ lattice QCD to judge whether chSB or conformal

§5 Summary

- 2 technical difficulties in evaluating Individual EVDs of massive chRMMs are overcome by Janossy Density formula + Quadrature method

- Individual Dirac EVDs of 2C QCD with $n_F = 4n$ staggered quarks, if the theory in χSB phase, are predicted from massive chGSE

- Chiral cond Σ of 2C QCD with $n_F = 2+(2+2+2)$ is determined by fitting $\mathrm{Spec}(\not{D})$ judged to be in the chSB phase (caution: $4/g^2 \leq 1.3$)

- feasible plan : determine whether the WTC candidates in sympl. class 2C QCD with $n_F = 8$ (stag.) , $n_{Ad} = 2$ (overlap) is chSB or conformal